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THE APPLICATION OF HYDRODYNAMICS TO IRRIGATION AND DRAINAGE PROBLEMS*

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The maintenance of soil fertility in irrigated regions depends in part on control of the movement of ground water† and of soil moisture. It is apparent that knowledge of the laws governing the flow of water in soils is essential to the effective control of its movement. Much experimental work has been done on the movement of water in soils but comparatively little use has been made of the fundamental laws of motion, either as a guide to experimental procedure, or in the interpretation of the experimental observations. Intelligent guidance to irrigation engineers, to managers of irrigation systems, and indeed to practical irrigators, in the proper use of irrigation water and in the solution of problems in the maintenance of soil fertility which arise from its improper use, is dependent on a knowledge of the laws which control the movement of water in soil. It is, therefore, important that experimenters studying movement of moisture in soils should ascertain to what extent the fundamental laws of motion of fluids advantageously may be applied to their problems.

Hydrodynamics is that branch of physics which deals with the motion of fluids. The term fluids includes both liquids and gases, but

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† The term ground water as here used refers to that water which completely fills the pore spaces in the soil; whereas the term soil moisture signifies the water which exists in the capillary form and which does not ordinarily completely fill all of the soil pore space.

the purposes of this paper require only a study of liquids. The science of hydrodynamics is essentially mathematical. To simplify the mathematics, it is customary in the preliminary analysis to assume that the fluids dealt with are frictionless and hence that they support only normal stresses. The fluids are also assumed to be continuous throughout the space considered.

In preparing this paper the writer has realized that some of his readers may find difficulty in following the mathematical methods employed. It is hoped, however, that those who have some knowledge of the elementary calculus will be able to understand the analysis. The most general case of the dynamics of deformable bodies involves tedious analytical difficulties, a consideration of which the purposes of this paper do not require. The basic courses in mechanics for engineers as usually taught are concerned primarily with rigid or semirigid bodies, and the college courses in hydraulics usually include only the applications of a few elementary principles of hydrodynamics. The law of conservation of energy is used in the development of Bernoulli's Theorem and this furnishes the primary hydrodynamical background in hydraulics.

The equation of continuity, although extensively employed in the solution of problems in hydraulics, is seldom either written or derived in its general form. Since hydraulics deals largely with non-compressible liquids and with one-dimensional flow, the use of Bernoulli's Theorem as the basis for the equation of motion, and the use of the equation of continuity in a restricted sense serves the needs fairly well. However, the more general hydrodynamical equations are serviceable in a complete study of the motion of ground water and of soil moisture.

In a study of capillary phenomena, soils investigators have generally used the term capillary attraction in a qualitative sense only. The measurement of this attraction in soils is not readily accomplished by direct methods. However, hydrodynamics makes it possible to measure resultant capillary attraction by indirect means, as is shown later. It is generally known that pressure differences from point to point in a liquid give rise to motion from points of high to those of low pressure where other forces such as gravity are not involved. For example, consider the flow of water in a level pipe line. Here the driving force is dependent upon the space rate of change of pressure. The influence of pressure differences on the flow of liquids has an important bearing on the study of the movement of capillary moisture.

In addition to a consideration of the fundamental hydrodynamical equations, the primary purpose of this paper is briefly to review the

efforts which have been made to apply the principles of hydrodynamics to the solution of soil-moisture problems and to indicate the possible outlook for future investigations. The preliminary analysis presented herewith is somewhat general in character and applies both to gases and liquids. The application of the more general equations to irrigation and drainage problems permits some restrictions which are pointed out as the work proceeds. Before developing the general equation of motion, and the equation of continuity in its general form, brief reference is made to the forces which influence the flow of water in open channels and in pipes.

Among irrigation and drainage engineers, it is common knowledge that the velocity of water flowing in open channels and in pipes is determined by two classes of forces, namely, the driving forces and the resisting forces. When a headgate is suddenly opened, permitting water to enter a canal, its velocity is accelerated because the driving forces, F_d, in a down-stream direction parallel to the canal bed at the outset are greater than the resisting forces, F_r, in the opposite direction. The resisting forces are dependent on the velocity since without motion, there is no friction and F_r is zero. For the velocities encountered in most open channels, F_r seems to vary directly with the square of the velocity, and, as the velocity increases, F_r is increased until finally it becomes equal to F_d. According to Newton's Second Law, letting "a" equal the acceleration in the direction of flow we may write for a given elemental volume of water of mass "m," the equation

$$ma = F_d - F_r \qquad \text{(1)}$$

It is evident from equation (1) that when F_r equals F_d, "a" equals zero. Using this basic equation, together with the fact that the resisting forces are a function of the velocity, Ganguillet and Kutter developed the well-known Chezy-Kutter formula, $V = C\sqrt{RS}$ in which,

V = velocity, usually designated in feet per second.
C = the Kutter coefficient which is a function of the roughness of the channel "n," and also of R and S.
R = the hydraulic mean radius in feet.
S = the slope.

Further attention is given this equation after a consideration of the basic elements of hydrodynamics. The fundamental hydrodynamical analysis which follows is not new to science but applications of the hydrodynamical equations and methods to irrigation and drainage problems are essentially new.

FUNDAMENTAL HYDRODYNAMICAL ANALYSIS

Consider the motion of an elemental volume of fluid in a frictionless medium under the influence of extraneous forces, such as gravity, and also of resultant pressure forces.

The acceleration of a moving particle at a point P, the coördinates of which are x, y, z, is found as follows: At the point P the particle has a velocity V and component velocities of V_x, V_y, and V_z. From P the particle moves to Q, the coördinates of which are $x+\delta x$, $y+\delta y$, $z+\delta z$, in the time δt. In going from P to Q the particle is "taking a step forward in space and also a step forward in time." The increase in the X component, V_x, of the velocity, V, is therefore the sum of the products obtained by multiplying the rates of increase of V_x in the x, the y, and the z directions by the displacements in the respective directions, plus the rate of increase with time multiplied by the interval of time. In mathematical language this change in the X component of the velocity is

$$\delta V_x=\frac{\partial V_x}{\partial x}\delta x+\frac{\partial V_x}{\partial y}\delta y+\frac{\partial V_x}{\partial z}\delta z+\frac{\partial V_x}{\partial t}\delta t \qquad \text{..........(2)}$$

As the change in velocity δV_x occurred in the interval of time δt, dividing equation (2) by δt gives the time rate of change of velocity, which is the acceleration.

Therefore, the X component of the acceleration is

$$a_x=\frac{\delta V_x}{\delta t}=\frac{\partial V_x}{\partial x}\,\frac{\delta x}{\delta t}+\frac{\partial V_x}{\partial y}\,\frac{\delta y}{\delta t}+\frac{\partial V_x}{\partial z}\,\frac{\delta z}{\delta t}+\frac{\partial V_x}{\partial t} \qquad \text{..........(3)}$$

But since by definition

$$V_x=\frac{\delta x}{\delta t},\quad V_y=\frac{\delta y}{\delta t},\text{ and } V_z=\frac{\delta z}{\delta t}$$

equation (3) becomes, after substituting values of $\delta x/\delta t$, $\delta y/\delta t$ and $\delta z/\delta t$

$$a_x=V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}+\frac{\partial V_x}{\partial t} \qquad \text{..........(4)}$$

Consider now the pressure forces acting on the elemental volume in a moving fluid. The resultant pressure on the elemental volume in the X direction may be found by reference to a rectangular parallelopiped within the fluid as illustrated in figure 1.

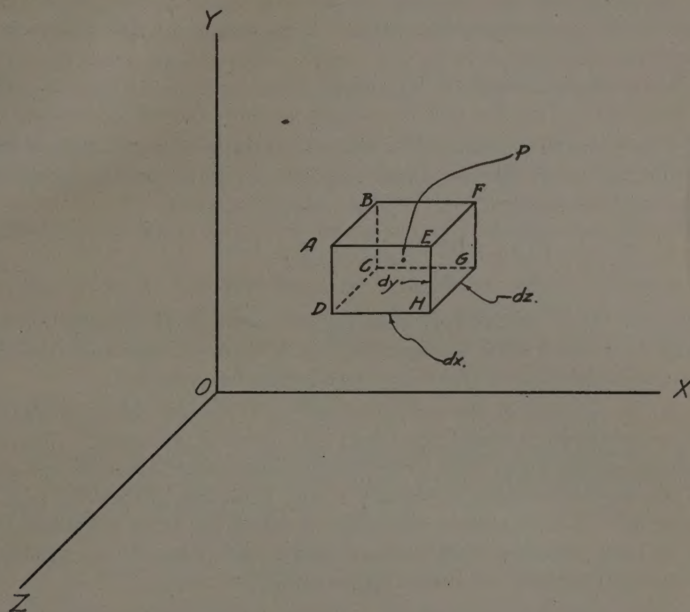

Fig. 1. An elemental volume within a moving fluid.

Let p be the pressure at the center of the elemental volume of which the lengths of the edges are dx, dy, dz. Let ρ be the density, or the mass in unit volume, and let V_x, V_y, V_z, be the component velocities at the point P. The average intensity of pressure on the face $ABCD$ is $\left(p-\frac{\partial p}{\partial x}\frac{dx}{2}\right)$, hence, the total pressure on the face $ABCD$ is $\left(p-\frac{\partial p}{\partial x}\frac{dx}{2}\right)dydz$ ....(a). Likewise the total pressure on the face $EFGH$ is $-\left(p+\frac{\partial p}{\partial x}\frac{dx}{2}\right)dydz$ ....(b); i.e., directed toward the yz plane. Hence, the resultant pressure in the x direction, found by adding (a) and (b), is $-\frac{\partial p}{\partial x}dxdydz$. Let F_x, F_y, and F_z be the components of the extraneous forces per unit mass of the fluid. The mass in the elemental volume is $\rho dxdydz$ and hence $F_x\rho dxdydz$, $F_y\rho dxdydz$, and $F_z\rho dxdydz$ are the resultant components of the extraneous forces on the elemental volume. Now as dx, dy, and dz may arbitrarily be

made very small, the elemental volume may be considered as a particle and hence by Newton's Second Law there results for the X direction,

$$\rho dxdydz\ a_x=\left(\rho F_x-\frac{\partial p}{\partial x}\right) dxdydz \qquad (5)$$

The right hand member of (5) is the sum of the resultant X components of the forces on the elemental volume. Dividing by the elemental mass and substituting from (4) the value of a_x gives

$$\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (6)$$

Equation (6) is the general equation of motion in the x direction for fluids in which friction is neglected. It is apparent that equations for motion in the y and z directions may be similarly derived.

In the analysis of the motion of perfect fluids two classes of motion are encountered, namely, rotational and irrotational motion. For the purpose of this paper only irrotational motion need be considered, and in this class of motion the velocity at any point may be derived from a potential. It is, therefore, desirable to define the term potential, and to consider somewhat fully the meaning of potentials, before applying the general equation of motion to irrigation problems.

The potential at any point P is defined as the negative line integral of the vector from some reference point to the point P, provided the magnitude of this integral has only one value. In mathematical language the potential is, therefore,

$$\Phi=-\int_{P_0}^{P}(E_xdx+E_ydy+E_zdz) \qquad (7)$$

where Ex, Ey, and Ez are the components of the vector E along the X, Y, and Z axes respectively.

It follows from the definition of a potential that in a gravitational field every point is characterized by a gravitational potential, in an electrical field by an electrical potential, in a capillary field or a moist soil by a capillary potential. The above potentials, together with some others, may be classed as a group of energy potentials since the line integrals are summations of force multiplied by distance. Under certain conditions, as briefly mentioned above, the velocity at every point is derivable also from a potential. The potential from which the velocity may be derived is called a "velocity potential." It is subject to all the mathematical operations of the energy potentials but differs in some properties as will be shown later.

The energy potentials as commonly used are further defined as the work done *on* existing forces in bringing unit mass from a specified reference point to any other point. For example, consider the work done on gravitational forces in moving unit mass from one point to another under the following conditions. Select a system of two bodies, one having a mass M and one having unit mass. When separated by a distance r, the attraction between these two bodies according to the inverse square law is $E = -\frac{kM}{r^2}$ where k is a constant, depending for its magnitude on the units used and considering r positive from M toward the unit mass. Therefore the work done *on* the gravitational forces in bringing the unit mass a differential distance dr directly toward the mass M is $dw = -\left[-\frac{kM}{r^2}dr\right]$.

The work W done on the gravitational forces in bringing the unit mass directly from the point P_0 to the point P distant respectively R_0 and R from M, therefore, is

$$W = -\int_{R_0}^{R} -\frac{kM}{r^2}dr = -kM\left[\frac{1}{R} - \frac{1}{R_0}\right] \quad \text{.........(8)}$$

Let the point P_0 be an infinite distance from M and equation (8) becomes

$$W' = -kM\left[\frac{1}{R} - \frac{1}{\infty}\right] = -\frac{kM}{R} \quad \text{.........(8a)}$$

In the gravitational region infinity has been selected as the reference point. Equation (8a), therefore, gives the gravitational potential at the point P, which is distant R from M, i.e., the work done on the gravitational forces in bringing unit mass from *infinity* to the point in question. A clear conception of the gravitational potential, which is determined by the familiar force of attraction, the magnitude of which is expressed by the inverse square law, helps one understand the meaning of the capillary potential.

The capillary potential at any point P, as here used, is defined as the work done *on* the capillary attraction in bringing *unit mass* of water from the level water surface to any point P within the capillary region. For example, select a column of soil, the water content of which is in equilibrium with the gravitational water in the reservoir, as illustrated in figure 2.

The system being in static equilibrium there is no movement of water in the soil column. Therefore, if we neglect friction, a very

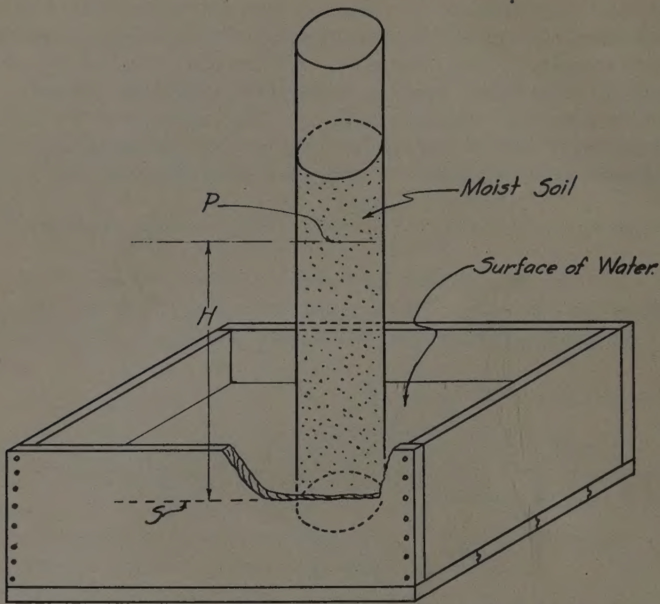

Fig. 2. A column of soil with the water content in equilibrium with the gravitational water in the reservoir.

slight upward impulse applied to unit mass of water at the level of the surface, S, will cause it to rise without having had work done on it to any point P distant H above S. The magnitude of the resultant capillary force as a function of H in bringing the unit mass from the water surface S to the point P is not known, whereas the gravitational force as a function of r in bringing unit mass from ∞ to the point P is known. But as the capillary water is in equilibrium under the action of both the gravitational and the capillary forces it is known that any gain in one form of energy by the unit mass must be equal in magnitude to a simultaneous loss in the other form of energy. The work done *on* the gravity forces, since the integration is in the positive H direction and against the force of gravity, is

$$W_g = -\int_O^H -g dH = gH^* \qquad (9)$$

* As H is small the variation in g with H is negligible.

The system being in equilibrium, W_g must be equal in magnitude and opposite in sign to the work ψ done on the capillary forces, because the sum of the two quantities of work must be zero. Therefore,

$$W_g+\Psi=0 \quad \text{or} \quad -\Psi=W_g=gH \quad \text{.........(9a)}$$

Equation (9a) makes it possible to compute the capillary potential Ψ at every point in a moist soil in which the capillary water is known to be in equilibrium with gravitational water. The capillary potential is a magnitude which characterizes every point in the moist soil.

In the light of the above consideration of potentials, it is desirable to see how the general equation of motion is related to potentials.

The Equation of Motion and Potentials: The vector components E_x, E_y, and E_z of equation (7) may represent component velocities, and hence, where Φ = a velocity potential,

$$\Phi=-\int(V_x dx+V_y dy+V_z dz) \quad \text{.........(10)}$$

Partial derivatives of Φ with respect to x, y, and z, give the equations:

$$\left.\begin{aligned}\frac{\partial\Phi}{\partial x}&=-V_x \quad \text{.........(a)}\\ \frac{\partial\Phi}{\partial y}&=-V_y \quad \text{.........(b)}\\ \frac{\partial\Phi}{\partial z}&=-V_z \quad \text{.........(c)}\end{aligned}\right\} \quad \text{.........(11)}$$

Knowing that the order of differentiation is immaterial in successive partial derivatives, there results:

$$\left.\begin{aligned}\frac{\partial}{\partial y}\left(-\frac{\partial\Phi}{\partial x}\right)&=\frac{\partial}{\partial x}\left(-\frac{\partial\Phi}{\partial y}\right) \quad \text{.........(a)}\\ \frac{\partial}{\partial z}\left(-\frac{\partial\Phi}{\partial y}\right)&=\frac{\partial}{\partial y}\left(-\frac{\partial\Phi}{\partial z}\right) \quad \text{.........(b)}\\ \frac{\partial}{\partial x}\left(-\frac{\partial\Phi}{\partial z}\right)&=\frac{\partial}{\partial z}\left(-\frac{\partial\Phi}{\partial x}\right) \quad \text{.........(c)}\end{aligned}\right\} \quad \text{.........(12)}$$

Substituting from equations (11) into equations (12), it is evident that:

$$\left.\begin{aligned}\frac{\partial}{\partial y}V_x&=\frac{\partial}{\partial x}V_y \quad \text{.........(a)}\\ \frac{\partial}{\partial z}V_y&=\frac{\partial}{\partial y}V_z \quad \text{.........(b)}\\ \frac{\partial}{\partial x}V_z&=\frac{\partial}{\partial z}V_x \quad \text{.........(c)}\end{aligned}\right\} \quad \text{.........(12a)}$$

Substituting from equation (12a), the values of $\frac{\partial V_z}{\partial y}, \frac{\partial V_x}{\partial z}$ in the general equation of motion (6) and also the value of $V_z = -\frac{\partial \Phi}{\partial x}$ from (11) there results

$$V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_y}{\partial x}+V_z\frac{\partial V_z}{\partial x}-\frac{\partial}{\partial t}\left(\frac{\partial \Phi}{\partial x}\right)=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \text{.......................(13)}$$

The negative term on the left of the equality sign in (13) may be expressed in the form $-\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial t}\right)$. Further, assuming that F_x may be derived from a potential, it is clear that $F_x = -\frac{\partial \Omega}{\partial x}$ where Ω is a potential due to extraneous forces characteristic of every point in the region. Substituting the above values in (13) shows that

$$\frac{\partial}{\partial x}\left(\frac{1}{2}V_x^2+\frac{1}{2}V_y^2+\frac{1}{2}V_z^2\right)-\frac{\partial}{\partial x}\left(\frac{\partial \Phi}{\partial t}\right)=-\frac{\partial \Omega}{\partial x}-\frac{1}{\rho}\frac{\partial p}{\partial x} \quad \text{..................(13a)}$$

Letting $V=$ the resultant velocity in (13a) or $V^2=V_x^2+V_y^2+V_z^2$ multiplying by ∂x and integrating with respect to space keeping time constant there results

$$\frac{1}{2}V^2-\frac{\partial \Phi}{\partial t}=-\Omega-\int\frac{\partial p}{\rho}+C \quad \text{..(14)}$$

But the constant of integration C is an arbitrary function of time, i.e., $C=f(t)$. Since precise determination of the velocity potential Φ can be made only with reference to a particular time, t, because the motion may not be steady, it is consistent to consider the C in (14) included in the $\frac{\partial \Phi}{\partial t}$ term and hence from (14),

$$\int\frac{\partial p}{\rho}=\frac{\partial \Phi}{\partial t}-\frac{1}{2}V^2-\Omega \quad \text{..(15)}$$

Equation (15) rests on Newton's fundamental law of motion with but two assumptions:

(1) That the extraneous force F_x may be derived from a potential.

(2) That the motion is irrotational.

Two further assumptions are now introduced, namely:

(3) That the density is constant as in water, and

(4) That the motion is steady or does not change with time.

Under assumption (3) $\int\frac{\partial p}{\rho}=\frac{1}{\rho}\int\partial p=p/\rho+a$ constant.

Under assumption (4) $\frac{\partial\Phi}{\partial t}=0$, since Φ at a particular point is constant with respect to time. Introducing these conditions in (15) there results

$$\Omega+\frac{p}{\rho}+\frac{1}{2}V^2=\text{constant} \qquad \text{(16)}$$

From equation (16) a number of relations of fundamental importance to irrigation and drainage may be derived.

The only extraneous conservative force which influences the flow of water in irrigation or drainage channels is gravity, and hence for unit mass in a channel at an elevation h, $\Omega=-\int_0^h -g dh = gh$ where the arbitrary reference point is called zero. Substituting for Ω the above value in (16) it follows that

$$gh_1+\frac{p_1}{\rho}+\frac{1}{2}V_1^2=gh_2+\frac{p_2}{\rho}+\frac{1}{2}V_2^2=\text{constant} \qquad \text{(17)}$$

Thus Bernoulli's equation (17), of fundamental importance to all branches of hydraulic engineering, including irrigation and drainage, is derived from Newton's Law of Motion.

Applying (17) to the flow through a submerged orifice, the pressures p_1 and p_2, being equal, disappear from the equation; and, as the initial velocity in the large channel of approach is so small that it may be neglected, therefore $2g(h_1-h_2)=V_2^2$ from which there results the well known Torricelli's theorem

$$V_2=\sqrt{2g(h_1-h_2)}=\sqrt{2gh} \qquad \text{(18)}$$

The equation of continuity, also fundamentally important in hydrodynamics, is based on the law of conservation of mass and is derived as follows: Consider as above an elemental volume whose edges are dx, dy, dz, as shown in figure 1, the volume being fixed in space in which fluid is moving. Let V equal the velocity of the fluid at the point P, which is at the center of the elemental volume. The average x component of the velocity in the face $ABCD$ is then $\left(V_x-\frac{\partial V_x}{\partial x}\frac{dx}{2}\right)$ and the mass of the flow across this face in unit time is $\left[\rho V_x-\frac{\partial}{\partial x}(\rho V_x)\frac{dx}{2}\right] dydz$. The average velocity in the face $EFGH$ is $\left(V_x+\frac{\partial V_x}{\partial x}\frac{dx}{2}\right)$ and the mass

of flow across this face in unit time is $\left[\rho V_x+\frac{\partial}{\partial x}(\rho V_x)\frac{dx}{2}\right]dydz$. Subtracting the latter mass from the former, the rate of gain in mass between the faces *ABCD* and *EFGH* is found to be equal to

$$-\frac{\partial}{\partial x}(\rho V_x)\ dxdydz \qquad \text{(a)}$$

Comparing similarly the flow across the face *CDHG* and *BAEF* it is found that the rate of gain is equal to

$$-\frac{\partial}{\partial y}(\rho V_y)\ dxdydz \qquad \text{(b)}$$

For the faces *CBFG* and *DAEH* the rate of gain is equal to

$$-\frac{\partial}{\partial z}(\rho V_z)\ dxdydz \qquad \text{(c)}$$

Adding equations (a), (b), and (c) gives the rate of change of mass in the elemental volume. Since mass can be neither created nor destroyed it is known that the sum of (a), (b), and (c) must be equal to the time rate of change of mass within the volume and hence it follows that

$$\frac{\partial}{\partial t}(\rho dxdydz)=-\left[\frac{\partial}{\partial x}(\rho V_x)+\frac{\partial}{\partial y}(\rho V_y)+\frac{\partial}{\partial z}(\rho V_z)\right]dxdydz \qquad \text{(d)}$$

Dividing (d) by *dxdydz* and transposing there results the equation of continuity in general form, i.e.,

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho V_x)+\frac{\partial}{\partial y}(\rho V_y)+\frac{\partial}{\partial z}(\rho V_z)=0 \qquad \text{(19)}$$

APPLICATION TO OPEN CHANNELS, PIPES, AND SOIL MOISTURE

The movement of moisture in soils is influenced by forces analogous to those which control the flow of water in open channels and pipes. The following brief analysis of well-known formulae in hydraulics, together with some analogies to a proposed soil moisture formula may be helpful in the study of moisture flow.

Open Channels.—The Chezy-Kutter formula $V = C\sqrt{RS}$ heretofore briefly mentioned, was derived from the general equation of motion for the condition of steady flow in which the driving forces are equal and opposite in direction to the retarding forces. Moreover, as indicated below, this formula rests on the fact that the velocity is determined by:

(1) the conductivity, which is dependent upon the form of the channel, and the roughness of surface, and,

(2) The rate of change of energy per unit mass in the direction of flow, or the component of the potential gradient parallel to the surface of the stream.

Consider a stream channel of uniform cross-section, the bottom of which makes an angle α with the horizontal (or sea level), as illustrated in figure 3. Select any length of canal l, from P_0 in the horizontal surface to any point P distant h above the horizontal. Obviously $h = l \sin \alpha$, and therefore from (9) the gravitational* potential φ at P is:

$$W_g = \varphi = -[-gl \sin \alpha] \qquad (20)$$

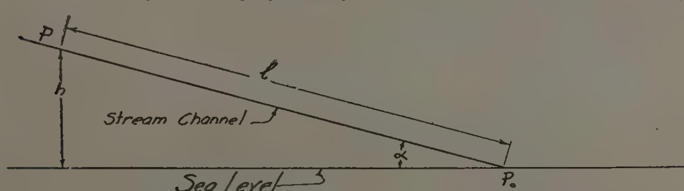

Fig. 3. Stream channel of uniform cross-section, making an angle "α" with a horizontal line.

The gravitational acceleration constant g is implicit in the Chezy-Kutter coefficient C and therefore $C = C_1\sqrt{g}$ where C_1 is a constant which is determined largely by the roughness of the channel. The velocity of water, according to the Chezy-Kutter formula, is

* For the gravitational potential as used in the following analysis, a point in the surface of a body of water or water table is taken as the reference instead of ∞ as in equation (8a). The integration is now opposite in direction to the force of gravity and the potential becomes positive.

$$V = C_1 \sqrt{R}\ \sqrt{gS} = C_1\sqrt{R}\ \sqrt{\frac{gh}{l}} \quad \text{.......... (21)}$$

The component of the potential gradient in the direction of the velocity is

$$\frac{\partial \varphi}{\partial l} = \frac{\partial}{\partial l}(gl \sin \alpha) = g \sin \alpha = g\frac{h}{l} \quad \text{.......... (22)}$$

Therefore, from (21) and (22)

$$V = C_1\sqrt{R}\ \sqrt{\frac{\partial \varphi}{\partial l}} \quad \text{.......... (23)}$$

The term C_1 of (23) involves the roughness of channel, the hydraulic mean radius R is the ratio of cross section area to the wetted perimeter and thus involves the form of the channel, and the expression $C_1\sqrt{R}$ is the conductivity, whereas the $\frac{\partial \varphi}{\partial l}$ is the component of the potential gradient along l.

Pipes.—The Weisbach formula commonly used for flow of water in pipes, in a similar manner, may be shown to consist of a conductivity factor and the component of a potential gradient. This formula is:

$$h_f = f\frac{l}{d}\ \frac{V^2}{2g} \quad \text{.......... (24)}$$

where:

h_f = loss of head in friction measured in feet.
f = a friction factor dependent on roughness of pipe.
l = length of pipe in feet.
d = diameter of pipe in feet.
V = the velocity in feet per second.
g = the gravitational acceleration constant.

Solving (24) for V^2 there results:

$$V^2 = \left(\frac{2d}{f}\right)\left(\frac{gh_f}{l}\right) \quad \text{.......... (25)}$$

and for pipes running full the hydraulic mean radius $R = \frac{A}{P} = \frac{d}{4}$ and hence from (25) for flow under pressure,

$$V = 2\sqrt{\frac{2R}{f}}\ \sqrt{\frac{gh_f}{l}} \quad \text{.......... (26)}$$

The factor $2\sqrt{\frac{2R}{f}}$ of (26) involves the form of pipe and its roughness and is the conductivity. The symbol h_f of (26) is analogous to the h of (21) and hence the factor $\frac{gh_f}{l}$ of (26) is the component of the potential gradient.

The above brief analysis concerning the Chezy-Kutter and the Weisbach formulae, it is hoped, will help make clear to irrigation engineers and investigators the purpose of, and the promise in, the application of hydrodynamics to other irrigation problems such as the movement of soil moisture.

Soil Moisture.—It is apparent by analogy from the foregoing illustration that soil moisture flow also is largely determined by two factors, namely:

(1) The conductivity of the soil, and

(2) The potential gradient within the moisture region.

As there are no restrictions on the direction of moisture movement, the flow will occur in the direction of the maximum rate of change of potential. However, the capillary stream is influenced by two potentials, namely: the capillary potential, Ψ, due to capillary pressure; and the gravitational potential, φ, due to gravity. We have, therefore, as a general equation for velocity of the capillary stream,

$$V = C[\nabla^*(\Psi+\varphi)]^m \qquad (27)$$

in which the conductivity factor is C and the potential gradient factor is

$$[\nabla(\Psi+\varphi)]^m.$$

For the capillary stream the exponent m is considered equal to unity for reasons given in the following section. The gradient of the potential due to gravity $\nabla\varphi$ being known, it is necessary only to measure Ψ at a few points, from which the component of $\nabla\Psi$ in the direction considered, may be determined. This possibility is further outlined after presenting a description of the method of measuring the capillary potential and a discussion of its relation to the soil moisture content.

There remains then the difficult but not insurmountable task of evaluating the conductivity, C, in (27) for typical soils under particular conditions of soil compactness, temperature, composition of soil water, moisture content and so on.

With the conductivity known, the measurement of the capillary potential, together with its relation to the moisture content would make possible the intelligent attack of irrigation and drainage problems.

Some of the more important of these problems, together with the methods of attack, are considered later. In the next few pages, the progress which has been made in the application of hydrodynamics to ground water and soil moisture is briefly reviewed.

* The symbol ∇ as here used is the gradient, i.e., the rate of change in the direction of the greatest rate of change.

GROUND WATER AND SOIL MOISTURE MOVEMENT INVESTIGATIONS BASED ON HYDRODYNAMICS

Ground Water.—The fruitfulness of dynamics in the study of the flow of heat, electricity, and perfect fluids has long been recognized. The more recent application of dynamics to the motion of ground water was made by Slichter[17] in 1897. Slichter's analysis is based on the equation of continuity (19) and on experimental observations of Darcy, Poiseuille,* and others. These investigators found, as quoted by Slichter,[17] that "the velocity of the flow of a liquid in a given direction through a column of soil is directly proportional to the difference in pressure at the ends of the column and inversely proportional to the length of the column." Mathematically expressed this law is

$$V = k\nabla p \qquad \text{(28)}$$

in which

v = the velocity
p = the pressure
∇p = the gradient of p, or the rate of change of p in the direction of the greatest rate of change.
k = a constant depending on the size of the soil grains, the soil porosity, the liquid viscosity, and the temperature.

The compressibility of water being very slight, its density ρ may be considered constant, and hence for water $\frac{d\rho}{dt} = 0$ and equation (19) reduces to

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \qquad \text{(29)}$$

Using Cartesian coördinates (28) may be expressed as:

$$\left.\begin{aligned} V_x &= k\frac{\partial p}{\partial x} \qquad \text{(a)} \\ V_y &= k\frac{\partial p}{\partial y} \qquad \text{(b)} \\ V_z &= k\frac{\partial p}{\partial z} \qquad \text{(c)} \end{aligned}\right\} \qquad \text{(30)}$$

* The experimental observation of Poiseuille is confirmed by Lamb[13] in a theoretical analysis of the flow of a liquid through a pipe of uniform section. (Hydrodynamics, 4th ed., § 331, p. 577.)

Substituting the values of V_x, V_y, and V_z from (30) into (29) and dividing by k there results

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=0 \quad \text{.......................................}(31)$$

Equations (29) and (30) are identical with Slichter's equations (4) and (5) of his chapter II.[17] Equation (31) is known as Laplace's equation and, like the equation of continuity (29) forms, in part, the basis of many important engineering analyses.

The fact that the "velocity potential," which satisfies (31), is proportional to the pressure in ground-water motion was first pointed out by Slichter. Moreover, Slichter made his excellent contribution to the investigation of ground water motion largely by pointing out the coincidence of the pressure function with the velocity potential (omitting the constant k) and by showing that the solution of any problem in the motion of ground water is dependent upon solving the differential equation (31).

Soil Moisture.—The first application of the analytical method to capillary action in soils was made by Buckingham[3] in 1907. He called attention to the fact that the movement of capillary moisture in any soil is dependent on its conductivity and on the driving force. Buckingham also pointed out, after proposing a general equation for measuring the flow of soil moisture, a formal analogy of this equation to the equations for flow of heat as measured by Fourier's law, and for the flow of electricity as measured by Ohm's law. Buckingham's equation is

$$Q=\lambda S \quad \text{.......................................}(32)$$

in which

Q="The capillary current density at any point, i.e., the mass of water which passes in one second through one square centimeter of an imaginary surface perpendicular to the direction of the flow."

S=The capillary potential gradient ($\nabla\Psi$), or "the amount by which the potential Ψ increases per centimeter in the direction of the current, by reason of the fact that the water content of the soil decreases in that direction."

λ="The capillary conductivity of the soil." The Q in equation (32) is numerically equivalent to the V of (28) hence we may write (32) in the form

$$V=\lambda\nabla\Psi \quad \text{.......................................}(33),$$

which is analogous to Darcy's experimental law[17] of flow for gravitational water.

Following Buckingham's work but little use was made of the fundamental laws of motion until 1920, when Gardner[7] briefly considered "the capillary potential and its relation to soil moisture constants." Very early in Gardner's soil-moisture studies he called attention to the fact that the capillary potential makes possible a new interpretation of soil-moisture constants, such as the hygroscopic coefficient, the moisture-holding capacity, the saturation constant, and the moisture equivalent. Each of these several constants really define equi-potential regions regardless of variation from one soil to another.

Concerning the definition of capillary potential, in the preliminary paper above referred to, Gardner says "It is perhaps quite immaterial where the zero potential is placed and also what convention is adopted as to the algebraic sign, although it is somewhat more in accord with modern usage to define the potential as the work done by the field forces in bringing unit mass from the point in question to infinity, and in such case the heat of vaporization should be added to Buckingham's potential and the negative sign should be used." However, for measurements of the capillary potential, which have not yet been published, Gardner has selected the water table as zero potential.

Gardner's later analysis of the dynamical problem with special reference to the movement of soil moisture appeared under the joint authorship of Gardner and Widtsoe[8] in 1921. The excellent analysis of these authors is based on the "assumption that the mean velocity of the water through the soil is proportional to the pressure gradient, or more generally, to the force per unit volume." That this assumption is supported both by fundamental analysis and by experiment is pointed out by the authors, as evidenced in part by the following statements:

"For bodies moving in response to conservative forces the resultant of the effective [external] forces is a measure of their acceleration in the direction of the resultant.* Where friction comes into play, however, this is not true; but a limiting velocity is soon reached when the frictional force becomes equal and opposite to the resultant of the impressed forces. For small velocities of such magnitudes as are encountered in the soil, this frictional force is directly proportional to the velocity. If, for example, water is forced through a small pipe of regular or irregular section, including the pores of a homogenous soil, the mean velocity is found from theory[13] and experiment[17] to vary directly as the pressure gradient† with a proportionality factor which involves the shape and size of the tube."

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* See also equation (1) and accompanying discussion in this paper.

† Some exceptions to this law were noted by F. H. King and recorded in U. S. Geol. Survey 19th Ann. Rept., pt. II. 61–294. *figs. 1–53. pls. VI–XVI.* 1898.

If the observation based on experiment and theory indicating that the frictional forces are directly proportional to the first power of the velocity be accepted as established law in soil moisture flow then equation (34) which is based on the Gardner-Widtsoe assumption may be derived with slight modification from the general equation of motion (6), as will shortly be demonstrated.

The Gardner-Widtsoe[8] equation is

$$V = K\rho\nabla\Phi^* \qquad (34)$$

where

V = mean velocity at a point in the soil.
K = a proportionality constant.
ρ = moisture density at a point.
Φ = the sum of three potentials, π, Ψ, and φ, where
π = potential due to hydrostatic pressure.
Ψ = potential due to capillary pressure.
φ = potential due to gravity.
∇ is a mathematical operator written thus

$$\nabla \equiv i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Referring again to the general equation of motion in frictionless media (6), it is apparent that for a steady state, or for acceleration so small as to be negligible, the left-hand member is zero. Applying the general equation (6) to the flow of soil moisture in any direction and in which there is friction and letting F_r = the resultant of the frictional forces on unit *mass* it is evident from (6) that

$$F - \frac{1}{\rho}\nabla p + F_r = 0 \qquad (35)$$

It is important to note that ρ as used in (35) and the analysis following is considered a variable. Conceive a given finite volume of soil as being composed (1) of a solid-soil phase, (2) a liquid-water phase, and (3) a gas (water-vapor-and-air) phase. If ρ is defined as the mass of phase (2), i.e., the liquid water, per unit volume of the space which is occupied by the three phases, then it is a variable. On the contrary, if ρ is defined as the mass of phase (2) per unit volume of phase (2), it would be substantially constant. The first definition given above, i.e.,

* The negative space rate of change of the energy potentials is work per unit mass divided by length, i.e., $-\nabla\Phi = \frac{\text{Force}}{\rho L^3} \times \frac{L}{L}$ = force per unit mass = F. Therefore, since $-\nabla\Phi = F$ and since there is in unit volume a mass of water, ρ, it follows that the magnitude of the resultant force per unit volume in the direction of V is $\rho F = \rho\nabla\Phi$, or the mass in unit *volume* times the force per unit *mass*.

the mass of liquid water per unit volume of space, is the one here used, and the applicability of the hydrodynamical equation of motion to the flow of soil moisture rests in part on the correctness of the concept of ρ as a variable. It rests also on the belief that the apparent discontinuities in the liquid-water phase may be ignored since in reality every particle of soil, or every small group of particles, is completely surrounded by capillary water and in this sense the liquid-water is continuous. It follows from the definition of ρ here used that V must signify the mean velocity of the liquid-water particles.

The quantity (mass) of liquid water, Q, that flows in unit time across a unit surface within the soil and normal to V, is given by the equation

$$Q=\rho V^{*} \qquad (36)$$

According to the above definition of ρ, it is evident that as ρ increases the capillary pressure decreases and therefore the capillary pressure is a function of the density ρ.

The only extraneous force influencing the moisture flow is gravity, and from the preceding discussion of potentials $F=-\nabla\varphi$ where φ is the potential due to gravity. Also, since the resisting forces per unit *volume* are directly proportional to the first power of the velocity, $\rho F_r=aV$ where "a" is a constant characteristic of the soil for a particular moisture content. Substituting these values of F and F_r in (35) there results

$$\rho\nabla\varphi+\nabla p=aV \qquad (37)$$

In the general case of soil water movement there are two kinds of pressure, namely:

p_c = capillary pressure or the negative pressure or tension due to curved surfaces, and

p_h = hydrostatic pressure.

The pressure p in (37) is equal to the sum of p_c+p_h ...... hence

$$aV=\nabla(p_c+p_h)+\rho\nabla\varphi \qquad (38)$$

Since $\nabla\Phi$ is force per unit *mass* and ρ = mass per unit volume, it follows that $\rho\nabla\Phi$ is force per unit *volume* and that

$$\rho\nabla\Phi \doteq \nabla(p_c+p_h)+\rho\nabla\varphi \qquad (39)$$

* If, however, ρ is defined as the mass of phase (2) per unit volume of phase (2), then its magnitude would be unity (approximately) with the C. G. S. units. Under this condition the quantity (mass) of liquid water, Q, that flows in unit time across a unit surface within the soil and normal to V, is given *numerically* by the equation:

$$Q=rV$$

Where V is the mean velocity of the liquid water particles, as above defined and r is the cross-section area of the liquid-water capillaries in unit surface within the soil and normal to these capillaries.

Hence (38) may be written by substituting from (39)

$$V = k\rho\nabla\Phi \quad \ldots \quad (40)$$

in which $k=\frac{1}{a}$. Equation (40) being the same as the Gardner-Widtsoe equation (34) indicates that the latter equation is in accord with the general equation of motion.

It is important to observe that Slichter's basic equation (28) for the flow of "gravitational" ground water and the Gardner-Widtsoe basic equation for the movement of soil moisture (34) are formally the same. The ∇p factor in (28) is equivalent to the $\rho\nabla\Phi$ factor in (34). Moreover, as shown by Gardner and Widtsoe, under the condition of complete saturation of the soil, at which time the capillary pressure is zero and the time rate of change of the mass per unit volume ρ is zero, their general equation of motion for capillary water, i.e., their formula (4) on page 221,[8] reduces to $\nabla^2 P=0$* which is identical with Slichter's equation (5), page 330, or number (31) in this paper.

Equation (II) of the Gardner-Widtsoe paper, as corrected in the footnote,* is the differential equation derived from the equation of continuity and the equation of motion for a steady state. After the introduction of certain tentative assumptions concerning the value of Ψ in terms of ρ, this equation reduces to

$$\nabla^2\rho+\frac{2\rho}{c}\nabla\varphi\nabla\rho=0 \quad \ldots\ldots (41)$$

which "is the most common condition met with in irrigation practice." Equation (41), together with others deduced from the general equation, including that for the condition of horizontal capillary flow under the influence of capillary forces only, and others for other conditions, are shown by Gardner to be in fairly close agreement with experimental data.

There is, however, in the movement of soil moisture an additional factor the influence of which, as yet, is not evaluated in the equations above reviewed, which may exert a significant influence, namely, the concentration of moisture or degree of saturation. Gardner and

* The Gardner-Widtsoe analysis on page 221[8] might be clarified a little by calling attention to the fact that the mathematical operator ∇ includes the unit vectors i, j, and k and is usually written

$$\nabla = i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}.$$

Further, in their case (2) the middle term of the last factor in the right-hand member should be $(\nabla\rho)(\nabla p)$ instead of $(\nabla p)^2$, thus making the corrected equation

$$\frac{\partial\rho}{\partial t}=-K[\rho\nabla^2 p+(\nabla\rho)(\nabla p)+2\rho\nabla\varphi\nabla\rho]$$

Widtsoe refer to this factor in the following language: "Where the soil is unsaturated and movement takes place in response to capillary forces, it is evident that the degree of saturation may become an additional factor, but in the absence of direct evidence that this has an appreciable effect upon the inherent moisture conductivity (K of equation 34), we may *temporarily* ignore the moisture concentration."

Buckingham[3] also recognized the importance of this factor and devoted considerable space to its consideration.

The design of suitable equipment and the development of methods for measuring the capillary potential has made possible a systematic study of the influence of the moisture concentration ρ on the moisture conductivity.*

MEASUREMENTS OF THE CAPILLARY POTENTIAL

Buckingham[3] measured the capillary potential by making direct readings of the height of rise of moisture in vertical soil tubes. As his method is fully described in available literature it is not further considered here.

For the methods described below I am indebted to Gardner and associates of the Physics Laboratory, Utah Agricultural College Experiment Station. The equipment now used by Gardner, a description of which has not heretofore been published, is shown in figure 4. It consists of a special U-shaped glass tube U; an inverted glass funnel F, to which is cemented a porous porcelain cup P; a bucket of soil S; and a quantity of mercury Hg. The inverted funnel is securely fastened to a connecting tube B, leading to one branch of the U-tube, by means of a tightly fitting vacuum rubber connection R. The connecting tube, B, is similarly fastened to the U tube by a vacuum rubber connection R′. The complete equipment is termed a capillary potentiometer.

The procedure in measuring the capillary potential is substantially as follows: The funnel F, after being cemented to the porous cup P with a mixture of hot asphalt, is attached to a suction pump to ascertain the maximum pressure which the cup will stand without leaking air. The cup is then filled with water, care being exercised to exclude air bubbles, and placed in the soil container as shown in figure 4 and connected at R. The connecting tube B is then filled with water, and the bucket, S, is filled with water up to the elevation of the plane Q, which causes the mercury in the small branch of the U tube to take the position

* An experiment concerning the influence of moisture content on conductivity is being conducted in the laboratory of the Physics Dept., Utah Agr. Exp. Sta.

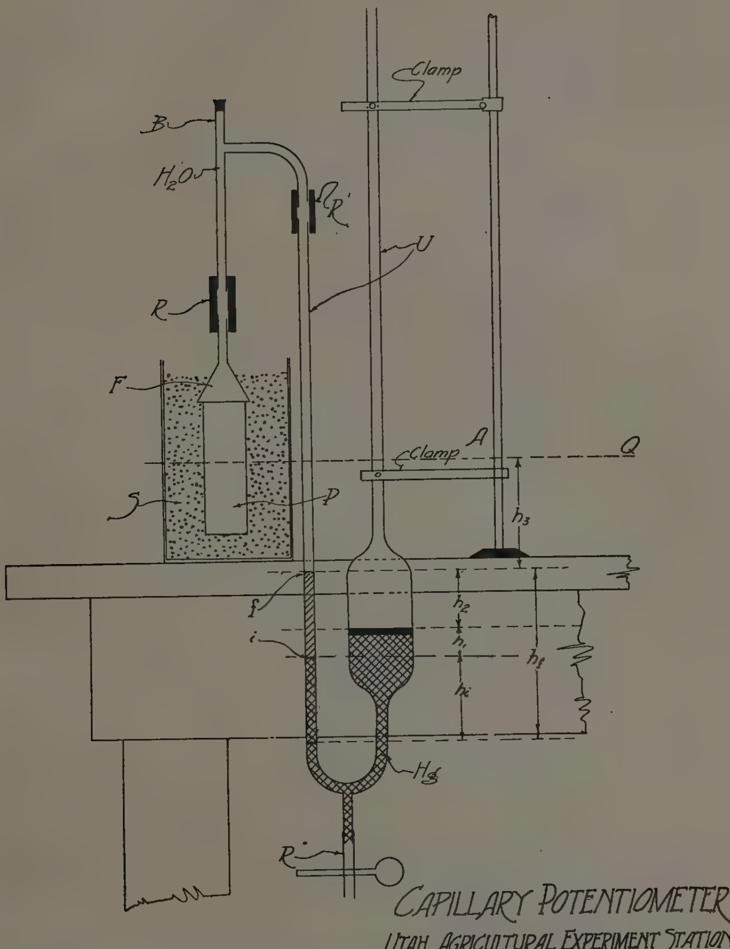

Fig. 4. The capillary potentiometer designed by Gardner, of the Utah Agricultural Experiment Station. This equipment is used in direct determinations. B, connecting tube. F, inverted glass funnel. P, porous porcelain cup. R, R′, R″, vacuum rubber connections. S, bucket of soil. U, U-shaped glass tube.

i. The water is then taken out of the bucket, and soil of the desired moisture content* is packed around the cup. The water is drawn by the soil through the walls of the porous cup. Thus the capillary forces do work against the force of gravity on the mercury by changing the mercury levels in the U tube from the initial position i, i.e., the position at which the mercury would rest if the porous cup were surrounded by water up to the plane Q, to the final position f, which represents the position after the soil has been packed about the cup and equilibrium has been reached.†

The capillary potential, Ψ, when measured in gram centimeters per gram, is numerically equal to the negative hydrostatic pressure measured in grams per square centimeter. Equation (9a) may therefore be written, by employing the above units,

$$-\Psi = H = p_f \text{ ....(numerically)........................} \quad (42)$$

where p_f = the hydrostatic pressure at a point equivalent to a distance H centimeters above a free water surface after equilibrium has been reached.

At the elevation of the plane Q, the hydrostatic pressure in the U tube of Fig. 4 is atmospheric at the beginning of the capillary potential measurement since the water surface in the soil bucket around the cup is then at the elevation Q. The initial hydrostatic pressure at elevation Q, when the mercury is at i, is therefore equivalent to zero capillary potential. After equilibrium has been reached and the mercury is at f, it is evident from Fig. 4 that

$$p_f + 13.6h_1 = 13.6(h_1 + h_2) + h_3 \quad \text{(a)}$$

Remembering that the pressure at elevation Q is atmospheric at the beginning of the measurement, it may be seen from Fig. 4 that when the capillary potential is zero

$$13.6h_1 = (h_1 + h_2) + h_3 \quad \text{(b)}$$

Subtracting (b) from (a) there results

$p_f = 12.6(h_1 + h_2)$ and substituting

in (42), it is apparent that

$$\Psi = -12.6(h_1 + h_2) \quad \text{(c)}$$

It is therefore necessary with this method only to measure the height of rise of the mercury column and multiply by (-12.6) to obtain the value of Ψ in gram centimeters per gram.

* The capillary potential of Greenville soil having less than 14 per cent moisture has not been measured as yet.

† The cross-section area of the U tube in the enlarged section is 30 times that of the small section, hence the lowering in the large section is ignored.

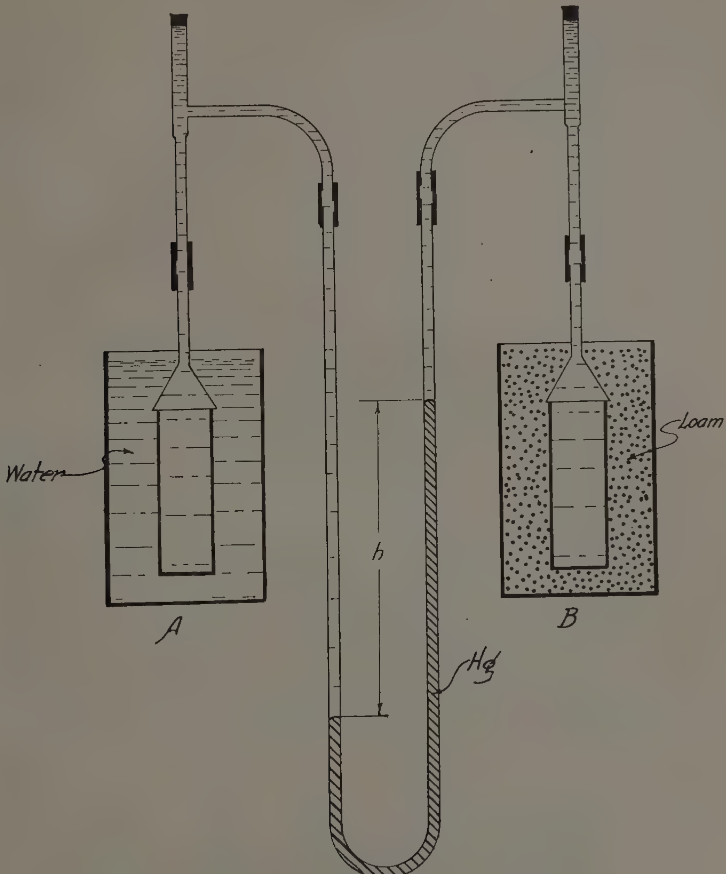

Fig. 5. Differential capillary potentiometer.

In order to measure the potential a differential method also has been developed. The equipment is illustrated in figure 5. It will be noted that bucket A has water outside as well as inside the porous cup. The potential as measured by h is then the absolute value for the soil in bucket B. If soil were placed also in A then h would measure the difference in Ψ between the soil in the two buckets.

Using the differential method just described, 9 buckets A, B, C, D, E, F, G, H, and I were arranged in series as shown in figure 6.

Bucket A was filled with water but contained no soil. The moisture percentage on the oven-dry weight basis ranged from 22.16 in B down

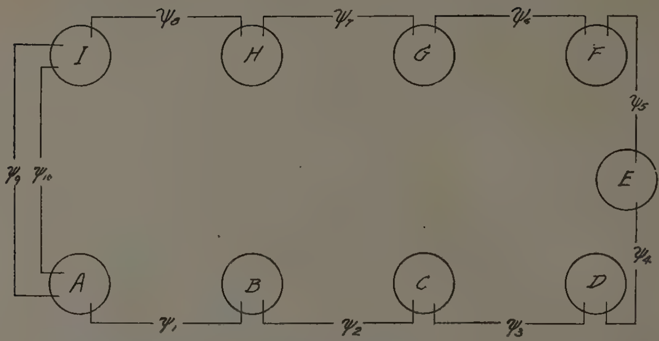

Fig. 6. Typical arrangement of buckets for measuring capillary potential by the differential method.

to 14.58 in Bucket I.* Measurements of the potential difference were made between buckets A and B, B and C, C and D, ... and so on by the potentiometers P_1, P_2, P_3, ... to P_{10}, and recorded as potentials Ψ_1, Ψ_2 . . . Ψ_{10}. The sum of the potential differences Ψ_1 to Ψ_8 should equal Ψ_9 or Ψ_{10} as measured by potentiometer P_9 or P_{10}, the latter two being duplicate measurements. At the end of an eleven-day test, the sum of Ψ_1 to Ψ_8 inclusive was 604 gm.-cm. per gm. and the average of Ψ_9 and Ψ_{10} was 614 gm.-cm. per gm. This is a satisfactory agreement. Further reference is made to the above measurements by the differential method in connection with the following data concerning the influence of the moisture percentage on the capillary potential.

RELATION OF THE CAPILLARY POTENTIAL (Ψ) TO THE MOISTURE CONTENT OF THE SOIL (ρ')†

That the curvature of soil moisture films is influenced by the moisture content of the soil is common knowledge among soils investigators. As the capillary potential Ψ at every point characterizes the film curvature at that point it is clear that the magnitude of Ψ is dependent in part on the moisture content ρ'. Therefore, $\Psi = f(\rho')$ for a given soil compactness, temperature, etc. If the relation $\Psi = f(\rho')$ were established for all conditions it would be possible to write, from (42)

$$V = k\rho\nabla[f(\rho') + \varphi] \qquad (43)$$

* Bucket E contained 24 per cent moisture, but, as one of the cups in this bucket leaked, this moisture content is rejected.

† The term ρ' signifies moisture percentage on the dry weight basis, hence $\rho = A_s\rho'$ where A_s = the apparent specific gravity of the soil.

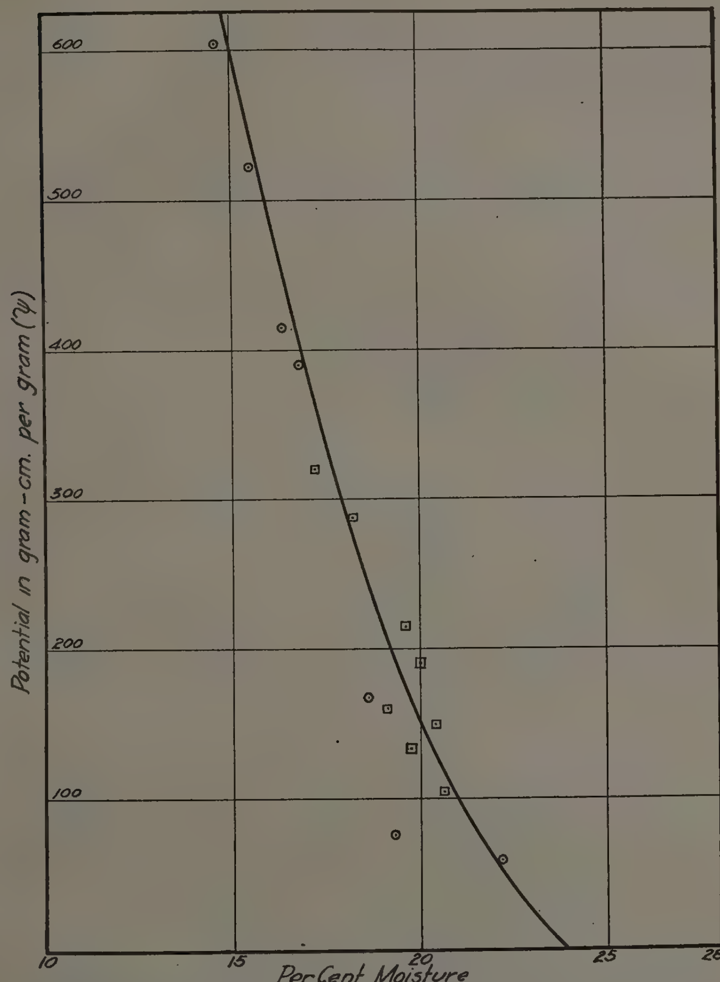

Fig. 7. Typical laboratory measurements of the relation of capillary potential Ψ to the moisture percentage ρ'.

TABLE 1

RELATION OF THE CAPILLARY POTENTIAL Ψ TO THE MOISTURE PERCENTAGE ON THE DRY WEIGHT BASIS ρ' AS INDICATED BY DIRECT METHOD MEASUREMENTS

A	B	C	D	E	F	G	H
Number of bucket	Potentiometer number	Depression of Hg in capillary tube with porous cup in water	Elevation of Hg in capillary tube when porous cup is in soil	Total height of rise of Hg in the capillary tube	Average total height for 3 potentiometers	Capillary potential in gm.-cm. per gram $=12.6(h_1+h_2)$	Moisture percentage based on dry weight of soil
		h_1	$+h_2$	h_1+h_2	$\frac{h_1+h_2}{3}$	Ψ	ρ'
6	a b c	17.0 15.9 17.3	8.6 9.1 8.0	25.6 25.0 25.3	25.3	319	17.2
2	a b c	17.4 16.7 14.8	5.8 6.3 7.7	23.2 23.0 22.5	22.9	288	18.2
7	a b c	6.2 5.3 6.3	6.9 6.8 6.6	13.1 12.1 12.9	12.7	160	19.1
3	a b c	12.9 11.5 10.6	4.4 5.4 6.2	17.3 16.9 16.8	17.0	214	19.6
8	a b c	2.4 2.3 1.1	8.5 7.7 9.2	10.9 10.0 10.3	10.4	131	19.8
1	a b c	10.2 10.4 7.8	5.4 5.1 7.4	15.6 15.5 15.2	15.4	194	20.0
4	a b c	5.8 4.7 7.5	6.0 6.9 4.7	11.8 11.6 12.2	11.9	150	20.4
5	a b c	2.3 0.9 2.5	5.9 7.1 6.2	8.2 8.0 8.7	8.3	105	20.6

From (43) the driving force causing the flow of soil moisture may be determined directly by measuring the space rate of change of ρ' from point to point in the soil.

Some typical laboratory measurements of the relation of Ψ to ρ' are presented in table 1 and also in figure 7. Of the 15 points plotted in figure 7, the eight points inside of the small squares were determined by the direct method of measuring capillary potential first described. The seven points within the small circles were determined by the differential method. Both tests were made with the Greenville soil. To determine the moisture percentage, ρ', all of the soil in each bucket was oven-dried.

Figure 7 indicates that the rate of change of the potential with the moisture content $\frac{d\Psi}{d\rho'}$, is comparatively high for low moisture percentages and low for relatively high moisture percentages. The figure suggests that the potential is high when the moisture content is low and that for large moisture contents, as would be expected, the potential is low.

As the capillary potential is zero at the water table, where the soil is completely saturated, it appears from figure 7 that the potential decreases relatively slowly as the moisture content is increased from 22.2 per cent to the saturation point which is probably well above 25 per cent. A statistical analysis of 84 laboratory determinations of the relation of Ψ to ρ', made by Gardner but not yet published, seems to indicate that over a certain range this relation may be represented by an equilateral hyperbola of the form

$$(\rho' - a)(\Psi + b) = c^{*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44),$$

in which a, b, and c are constants that may be evaluated from experimental observations. Moreover, this relation is apparently in accord with natural conditions. It seems, for example, that Ψ may become very large for a moisture content near the wilting point. According to (44) when Ψ is infinite $\rho' = a$ and when ρ' is infinite $\Psi = -b$. That equation (44) precisely represents the (Ψ, ρ) relation is not yet established, and therefore the data presented in table 1 and figure 7 should be interpreted as illustrating the trend of variation of Ψ with ρ'. Further reference is made to the data presented in figure 7 in connection with a study of the field capacity of soils for irrigation water, results of which are presented in figure 14.

* The maximum Ψ as yet measured is slightly over 600 gm.-cm. per gm. and the minimum moisture content is a little over 14 per cent.

SOME APPLICATIONS TO IRRIGATION AND DRAINAGE PROBLEMS

It is not the intent of this paper to enumerate all the ways in which hydromechanics may advantageously be applied to irrigation and drainage problems, nor to evaluate coefficients for the soil moisture velocity equation $V = k\rho\nabla\Phi$. On the other hand, the purpose is to call attention to the fact that use can be made of knowledge concerning equipotential regions and capillary potential measurements in a study of water capacity of soils and of moisture conditions above a high water table.

Irrigation and drainage are simply means of controlling the water content of arable soils for purposes of producing economical crops, the purpose of irrigation being to maintain in the soil an adequate moisture content, and that of drainage to prevent the occurrence of excessive amounts of water. The ideal way to irrigate is to supply water at the same rate as it is needed by plants, but such procedure is impracticable. It is therefore necessary to use the soil as a water reservoir in which there may be stored in a form available to plants the amount of water needed during the time between irrigations. The necessity for such storage, the difficulty in determining the capacity of soils for water, the inherent dread of drought among farmers in an arid climate, the great variation of soil texture and structure, together with other important factors, have led to the application of excessive amounts of irrigation water. Excessive irrigation, seepage from canals, and percolation from high lands to low lands in western valleys have brought about a rise in the water table which has rendered large areas of the best land either partially or wholly unproductive without artificial drainage. During the early years of irrigation, because of the very dry conditions of virgin soil and the great depth to the water table, the ultimate seriousness of wasteful use of water is not apparent. However, after many years of irrigation, when water has become more valuable, when the virgin aridity of the soil has been removed, and the water table has risen to elevations which endanger plant life, it becomes very necessary to have dependable information concerning soil moisture control. The capacity of the soils for water, the movement of soil moisture and of nitrates and other soluble salts, and the relation of drainage to capillary phenomena become increasingly important where irrigation has been practiced many years. That the solution of these problems may be furthered by the application of hydromechanics is pointed out below.

Capacities of Soils for Water.—Many advantages are gained by storing in the soil from a single irrigation all of the water it will retain for plant use, and likewise there are many disadvantages in applying in a single irrigation more water than the soil will hold. Some methods of determining the water capacity of soils have heretofore been reported[10,11] and only those phases of the question not considered previously will be discussed here.

The precision of water capacity measurements rests in large measure on two important factors, which, in general, have not been sufficiently recognized by soil moisture workers, namely:

1. The time after flooding the soil which is selected as representing the moisture capacity, and

2. The depth of the water table, or the reference point for the capillary potential.

The importance of these two factors, in reality, is due to the fact that under the natural conditions encountered in irrigation the soil moisture is seldom at rest. It is very difficult to obtain a condition of equilibrium of moisture in field soils.

It is likewise difficult to ascertain whether or not the moisture in a soil is in a condition of equilibrium. During the period in which equilibrium of moisture conditions is being approached, the velocity of moisture movement is so low that it cannot easily be detected by direct measurement. Since there can be no hydrostatic pressure in unsaturated soils in which capillary pressures exist, it follows that the total potential Φ of equation (40) is equal to the capillary potential, Ψ, plus the gravitational potential, φ. The gradient of the gravitation potential, $\nabla\varphi$, is known to be g. If, therefore, several measurements of the capillary potential, Ψ, at different elevations show that the gradient of the capillary potential $\nabla\Psi = -g$, then the moisture region considered is an equipotential one and that there can be no vertical moisture movement because the driving force is zero. The need of some satisfactory method of detecting the existence of equilibrium of moisture was urgently felt in 1919 in connection with some water capacity studies of soils under natural field conditions at the Utah Experiment Station. The results of these experiments are of interest in showing the difficulty of detecting without the aid of capillary potential measurements at what time the moisture reached a condition of equilibrium. They indicate also that when the condition of equilibrium is approached, after *excessive* irrigation of soils having uniform texture, the moisture content increases as the depth of the soil increases. A non-technical report[11] of these water capacity experiments was published in 1922. A brief statement of the plan of the experiments taken from this report is given below:

"To remove all doubt concerning *completeness of saturation* and also to remove the influence of the growing crop, the authors prepared three rectangular basin plats to which excessive amounts of water were applied. Each plat was 38 feet long and 33 feet wide. Around these plats levees about two feet high were built with soil taken from outside of the plats; thus, the soil in the plats was left undisturbed. The plats were numbered A, B, and C. Samples of soil were taken to ascertain the moisture content before irrigation, after which plat A was given a 12-inch irrigation, plat B a 24-inch irrigation, and plat C a 36-inch irrigation.

"The borings for moisture samples were made to a depth of 12 feet and the moisture determinations were made in the laboratory by the usual methods, the results being recorded in per cent of the weight of the dry soil."

The soil is a deep uniform loam having a water table about 50 feet below the surface. Widtsoe and McLaughlin[19] have published a detailed statement of the chemical and physical properties of the soil.

Observations concerning the movement of capillary water in the three plats, A, B, and C, are presented graphically in figures 8, 9, and 10. The data are reported in acre-inches of water per acre, for different depths of soil. Moisture determinations were made at irregular intervals from June 16 to October 11, 1919; there being 2,556 determinations, of which 1,476 were made in June, 468 in July, and 216 in August and September. The location of borings was systematically made, and stakes were placed in each hole as soon as the sample had been obtained and the excess disturbed soil had been replaced. On each stake the date of sampling was marked, thus avoiding duplication in the location of borings.

On June 16, immediately after the irrigation water disappeared from the surface of the soil, a heavy straw mulch from 8 to 10 inches deep was spread over the surface of each plat. To determine the loss of water through the straw, an evaporimeter pan 12″ by 20″ was filled with soil of about the same moisture content as that in the plat and was placed under the straw in plat A with its surface flush with the ground surface. From August 2 to 26, the pan lost 1,294 grams of water, which is equal to 0.838 cm. depth, or 0.035 cm. a day. Measurement of the decrease in moisture content of the upper 6 feet of soil from June 16 to August 22, after deducting the water evaporated, shows a loss from plats A, B, and C of 0.58, 0.64, and 0.71 cm. respectively, in 24 hours. These measurements are based on six borings in each plat and six samples in each boring.

The averages of the samples for each of the plats on the various dates on which moisture tests were made is presented for three different depths of soil in the curves of figure 8. It will be noted by the rapid decline of the upper curves of figure 8, each point of which is based on 72 moisture determinations, that large amounts of capillary water passed below the 12-foot level in plats B and C shortly after the irrigation. It appears that in plats B and C, and in the upper 6 feet of plat A, the moisture continued to move downward from June 16 to August 22. From June 16 to June 30 in plat A there was a significant increase in the moisture content from 7 to 12 feet in depth, but from July 5 to August 22 there seems to have been a downward movement even from this depth, as the total loss of moisture is greater than the evaporation loss.

The curves for the entire 12-foot depth show that equilibrium was closely approached, if not actually reached, on August 22, because after this date there was practically no further loss. The small gain in October was due to a rainfall of 1.36 inches early in the month.

In figure 9 the loss of moisture for each plat is compared in four depth zones of soil, namely, from 0 to 3 feet, 4 to 6 feet, 7 to 9 feet, and 10 to 12 feet inclusive. The notable features shown by this figure are that in the upper 3 feet of soil, plat A, which received a 1-foot irrigation, was almost as fully wetted at the outset as plat C, which received a 3-foot application. In the second 3 feet of soil the water content in plat A was appreciably less than that in plats B and C, during the first 10 days after irrigation. In the third and fourth 3-foot sections the differences are much greater in magnitude and of longer duration. To illustrate: in the fourth 3-foot section the moisture content in plat A was lower than that in plats B and C until after August 22.

Figure 10 compares the moisture content of each of the four 3-foot soil zones within each plat. It will be noted that during the early part of the period there were large differences between the moisture content in each of the four zones in plat A, smaller differences in plat B, and still smaller differences in plat C. That the moisture content of the first 3-foot zone remained the highest in all of the plats during the major part of the period is apparently due in part to differences in soil texture. Moisture equivalent determinations in each of the upper 12 feet for each plat show that the soil becomes somewhat coarser in texture with the depth.

It has commonly been assumed,[11,19] that the moisture content of the soil a few days after irrigation represented the moisture capacity and that but little moisture movement occurred after the relatively rapid movement of the first few days. Figures 8, 9, and 10 show the difficulty

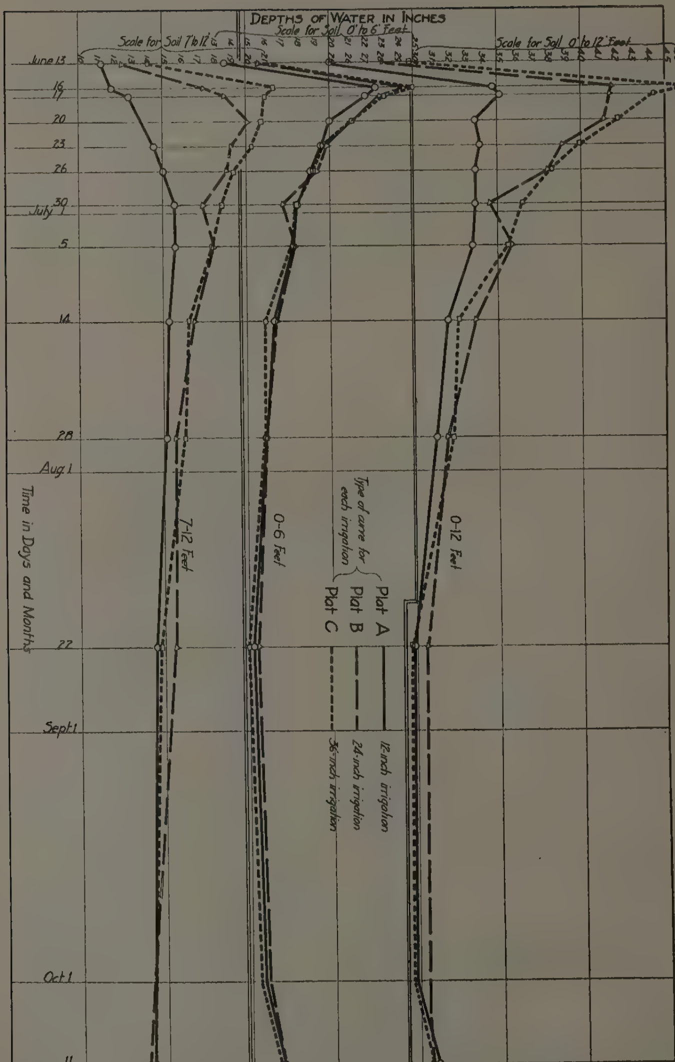

Fig. 8. Curves comparing the time rate of change in the amounts of water contained in the same depths of soil after the application of three different amounts of irrigation water. Results are expressed in inches depth of water in each of the three depths of soil considered.

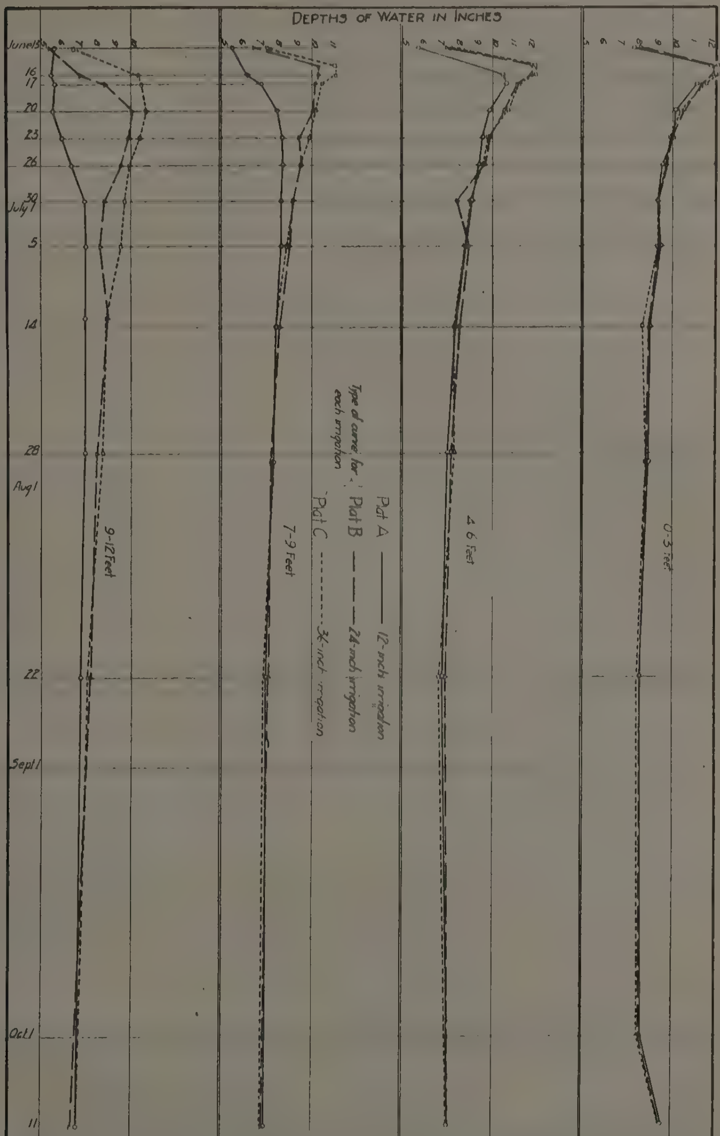

Fig. 9. Curves comparing the time rate of change in the amounts of water contained in the same depths of soil after each of three different irrigations. Results are expressed in inches depth of water in each of the four depths of soil considered.

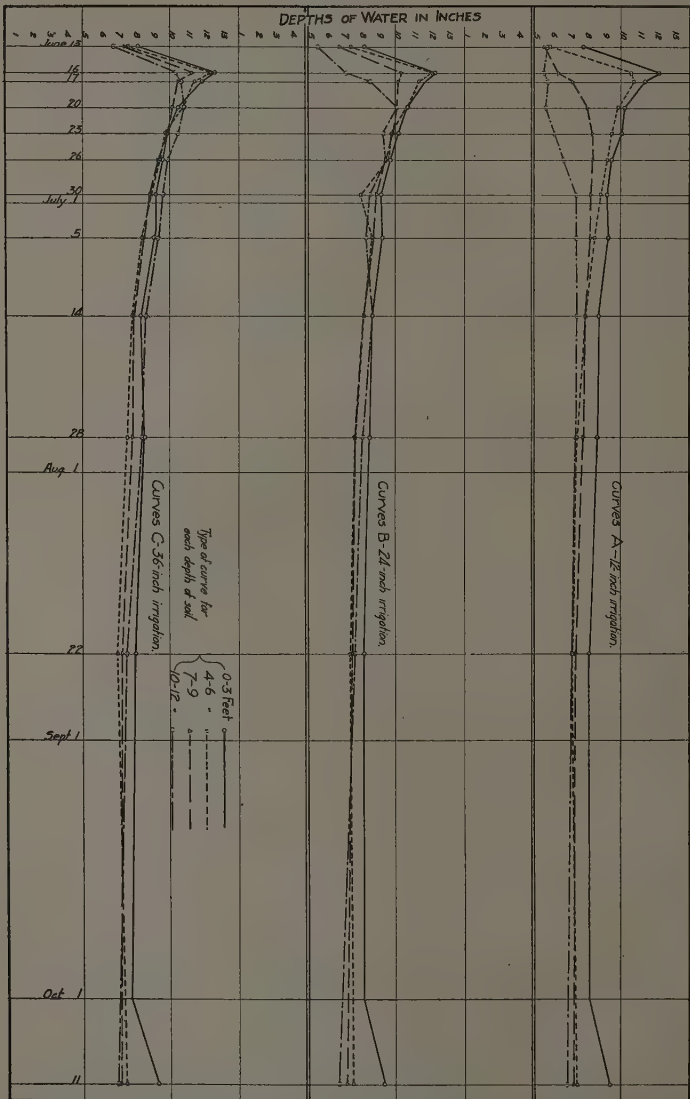

Fig. 10. Curves comparing the time rate of change in the amounts of water contained in four different depths of soil, after the application of three different amounts of irrigation water. Results are expressed in inches of water in each three feet of soil.

of selecting a particular time after irrigation as representing the maximum capacity. August 22 is apparently the only date that may be selected as representing the maximum without being arbitrary and even this date may not be chosen without the support of a mathematical analysis of all the experimental data. Such an analysis is reported after considering the vertical distribution of moisture in the soil of the three plats at different times after irrigation as shown in figures 11, 12, and 13. The moisture percentages are plotted in the fourth quadrant with the positive abscissa representing moisture content and the negative ordinate representing depth of soil in feet below the surface. In each of the figures there are five curves: (a) representing the moisture content before irrigation, (b), (c), and (d) representing the moisture content at different periods after irrigation, and (e) representing the Briggs-McLane moisture equivalent.

A comparison of the curves (a) and (b) in each of the three figures shows the influence of the amount of water applied to the soil on the depth of penetration shortly after irrigation. Curve (b) of figure 11 shows that only the upper 4.5 feet of soil had been fully moistened two days after the 12 inches of water had been applied. In figures 12 and 13 curves (c) indicate that the twelfth foot of soil was fully moistened 5 days after irrigation, at which time it contained much more moisture than it did 68 days after. In figure 11, on the other hand, the soil below a depth of 8.5 feet held appreciably more water 69 days after irrigation than it did 2 days after. Curves (d) show that 68 days after irrigation each of the plats held approximately the same amounts of moisture. Moreover, in the upper 6 feet of plats B and C the soil held no more moisture 68 days after irrigation than it did before.

Remembering that evaporation losses were reduced to a minimum and noting the heavy loss in moisture from June 16 to August 22 by downward flow it is of interest to inquire if the moisture content had reached equilibrium on August 22.

To do this Pearson's method of moments was employed to evaluate the constants in the equation

$$\rho' = \rho'_E + Ae^{-Bt} \qquad \text{.......................................(45)}$$

in which

ρ' = the moisture percentage at a given depth of soil at any time t.

ρ'_E = the moisture percentage at equilibrium at any depth.

e = the base of natural logarithms.

t = the average time in days measured from the date of first sampling after irrigation.

A and B are constants to be evaluated from the results of the moisture determinations at different dates.

PLAT A

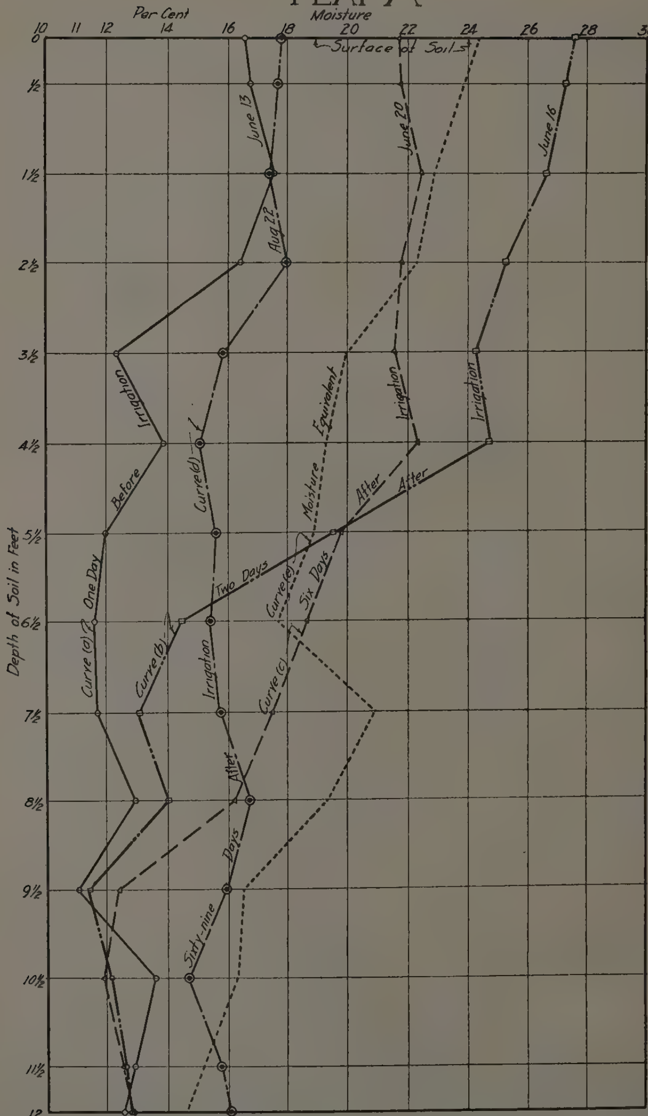

Fig. 11. Distribution of moisture in soil at different periods after a 12-inch irrigation.

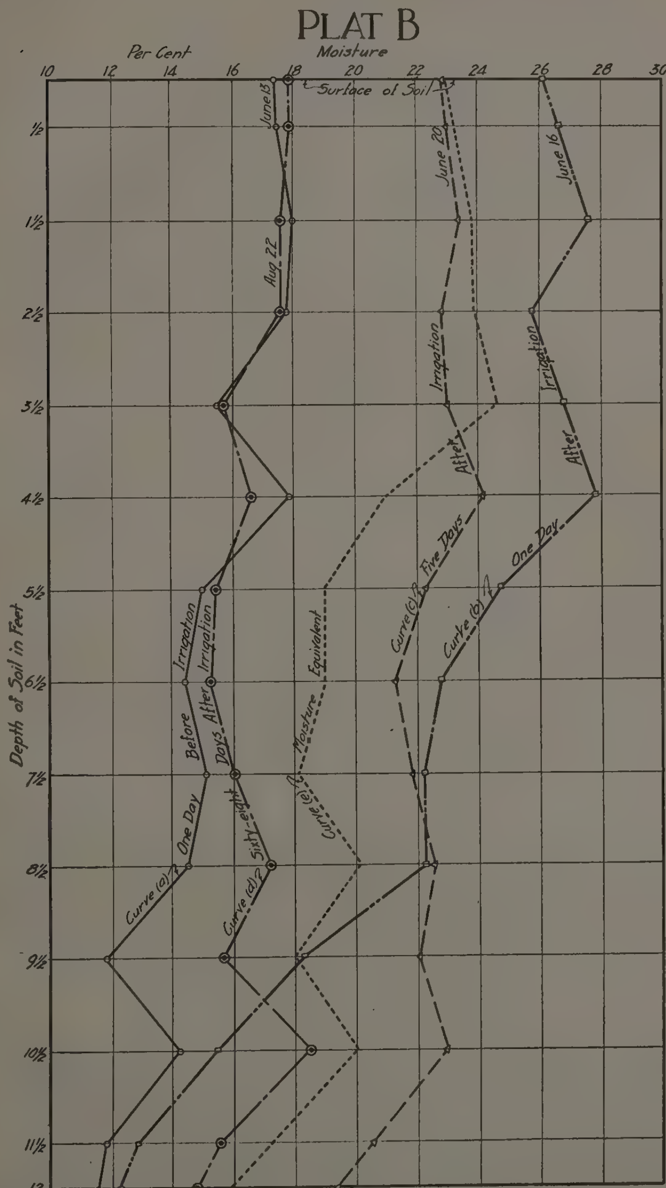

Fig. 12. Distribution of moisture in soil at different periods after a 24-inch irrigation.

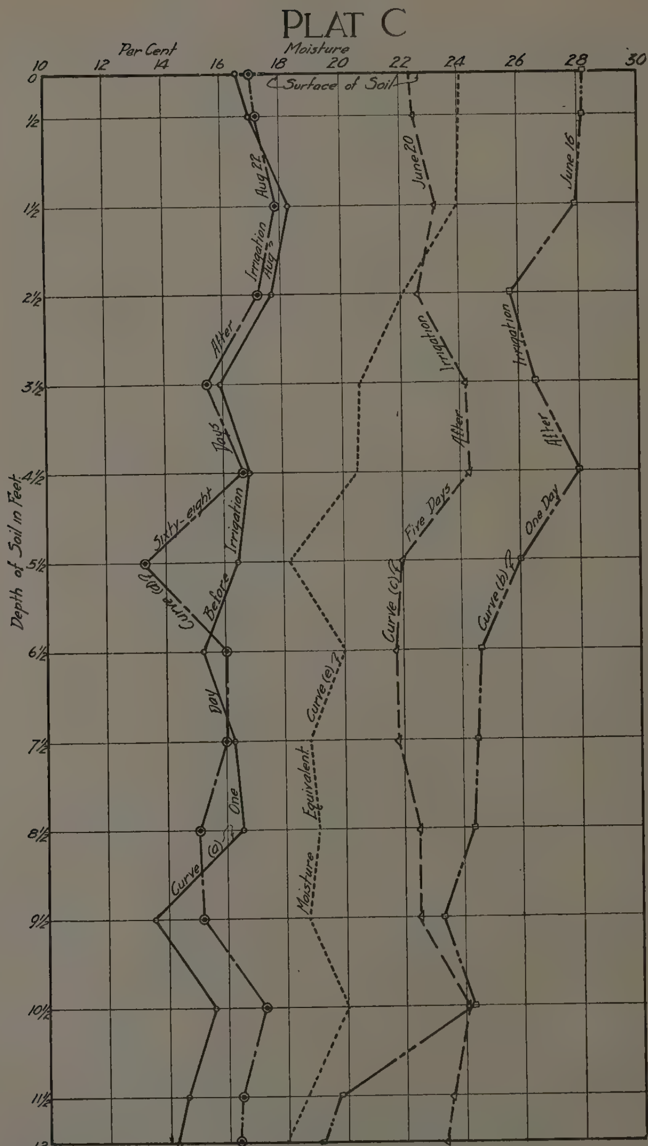

Fig. 13. Distribution of moisture in soil at different periods after a 36-inch irrigation.

Equations of the general type of (45) have been found to express physical relations in many natural phenomena. This type of equation seems to apply to the rate of moisture loss. According to (45), when time is infinite $\rho' = \rho'_E$, which means that the moisture in a field soil from which evaporation is prevented and in which no crops are growing in time will reach a condition of equilibrium. Finding the value of ρ'_E should, therefore, give the moisture percentages at equilibrium at the several depths of soil. It is also evident from (45) that when t equals zero $\rho' = \rho'_E + A$. The analysis by the method of moments was based on the samplings from June 16 to July 14.* A comparison of the magnitude of the moisture content at approximate equilibrium in the upper 9 feet of soil in plat C, as determined by equation (45), to the moisture content on August 22 is given below:†

Depth in feet	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5
Moisture content at equilibrium ρ'_E computed	17.3	18.1	17.8	16.7	17.6	14.2	17.8	15.7	12.7
Moisture content on ρ' August 22	16.9	17.8	17.2	15.5	16.6	13.3	16.2	16.0	15.0

The differences between ρ'_E and the ρ' determined on August 22 are probably not larger than the experimental error. The analysis by which the determinations of ρ'_E were made was based on only 7 sets of moisture determinations and hence should not be interpreted as definitely establishing the equilibrium of moisture content. Since, however, the decrease of ρ' between July 28 and August 22 was comparatively small as shown in figures 8, 9, and 10, it is likely that the moisture content on the latter date closely approached the equilibrium. Had the field capillary potentiometer been perfected at the time these water-capacity experiments were conducted it would have been possible to obtain more direct and more dependable information concerning the moisture condition at equilibrium.

If the August 22 determinations do represent the moisture content at equilibrium, then ρ'_E should increase with depth of soil provided the soil were uniform in texture. The moisture equivalent determinations in each plat to a depth of 12 feet indicate a significant increase in coarse-

* The samplings of July 28 and August 22 were omitted because the large period of time between these samplings gave them undue weight in the analysis.

† The influence of soil texture on the moisture content at equilibrium with change of depth is considered in the following pages.

ness of texture with depth. The influence of this variation in texture has been eliminated to a large extent by the following procedure:

Let $\rho'_1, \rho'_2, \cdots \cdot \rho'_{12}$ be the actual moisture percentages in the respective depths on August 22.

Let $e_1, e_2, \cdots e_{12}$ be the moisture equivalent in the respective depths.

Let e_m be the mean moisture equivalent for the 12 feet.

Let $\rho'_{c1}, \rho'_{c2} \cdots \rho'_{c12}$ be the corrected moisture percentages in the respective depths on August 22.

Then

$$\rho'_{c1} = \rho'_1 \frac{e_m}{e_1}; \quad \rho_{c2} = \rho'_2 \frac{e_m}{e_2} \cdots\cdots\cdots\cdot \rho'_{c12} = \rho'_{12} \frac{e_m}{e_{12}}$$

The corrected moisture percentages for each of plats A, B, and C on August 22 are plotted in figure 14, the points for plat A being represented by circles, for plat B by triangles and those for plat C by squares. The curve of figure 14 represents the average corrected ρ'_E for the three plats. It is evident from this figure that, had the Greenville soil been uniform in texture to a depth of 12 feet, the moisture percentages 68 days after irrigation would have increased significantly with increase in depth. It is important to note that the distribution of the moisture with depth, as shown in figure 14, represents a condition that might have been qualitatively predicted from the capillary potential analysis, namely, that as the depth increases from the surface of the soil toward the water table, the numerical magnitude of the capillary potential decreases and consequently the moisture percentage of the soil must increase. The capillary potential at the surface of the plats A, B, and C cannot be computed with precision since the distance from the surface to the water table is unknown. However, letting Ψ_a be the capillary potential ½ foot below the surface where the soil samples were taken to represent the first foot and Ψ_b be the capillary potential 11½ feet below the surface, where the samples were taken to represent the 12th foot, the difference in capillary potential at the two points, according to equation (9a) is

$$\Psi_a - \Psi_b = -(1 \times 11 \times 30.5) \cdots\cdots \rightarrow = -335.5 \text{ gm.-cm. per gm.*}$$

This capillary potential difference was accompanied by a variation in the moisture content of 4 per cent, which is the decrease in moisture between the 11.5-foot depth and the 0.5-foot depth. Assuming that

* Assuming the water table to be 60 feet below the soil surface, which is the depth to water in a neighboring well, it would follow that:

$$\Psi_a = -59.5 \times 30.5 = -1815 \text{ gm.-cm. per gm.}$$

$$\Psi_b = -48.5 \times 30.5 = -1479 \text{ gm.-cm. per gm.}$$

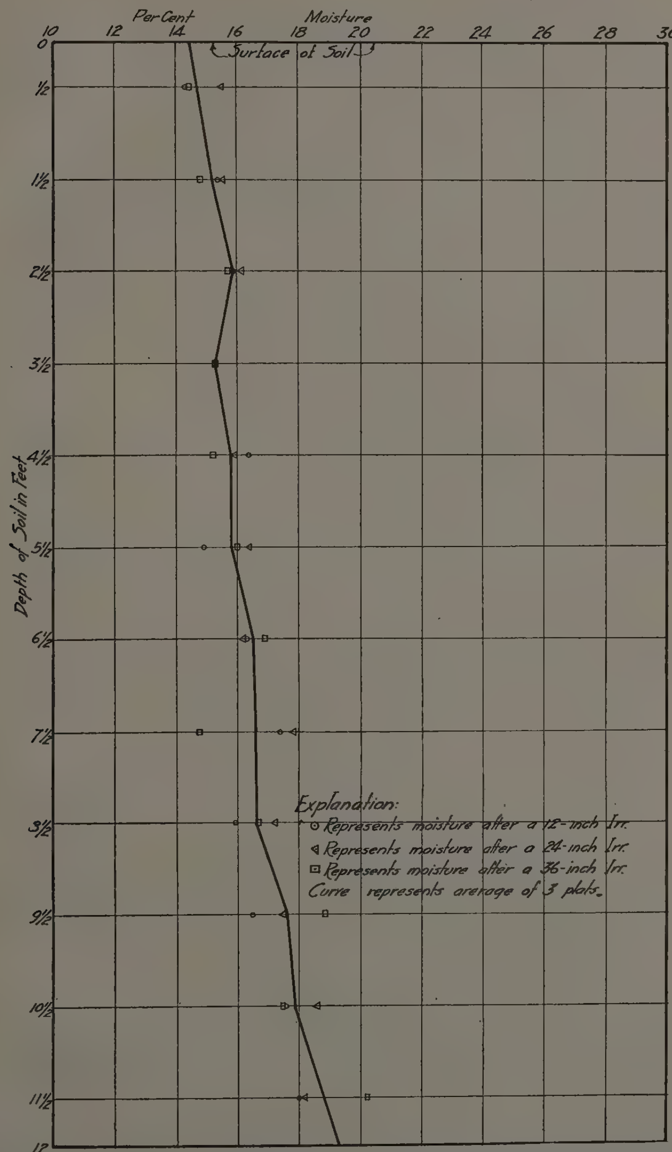

Fig. 14. Distribution of moisture in soil 68 days after irrigation as corrected for variation in soil texture, thus showing distribution that would have occurred in a soil of uniform texture.

the moisture content had reached equilibrium by this date, which was 68 days after irrigation, it is evident from the influence of change in the capillary potential on the equilibrium of the moisture content of the soil, as indicated in figures 7 and 14, that the depth to the water table influences appreciably the water capacity of the soil at equilibrium. Moreover, the recognition of the equilibrium of moisture content as an equipotential region (capillary and gravitational) is of itself sufficient basis for correcting the common belief[12,16] that the distribution of water in vertical soil columns at equilibrium is uniform. The theoretical basis for correction of this erroneous view is here supported by a study of the Greenville loam soil in the laboratory and in the field. Recent laboratory experiments by McLaughlin[16] also support the results of the above analysis.

A similar equilibrium soil moisture distribution will occur in a field of force other than the gravitational one. This has been experimentally verified[18] for a "centrifugal" force field in the laboratory of the Division of Irrigation Investigations and Practice of the University of California, Agricultural Experiment Station at Davis. Attention is directed particularly to figures 4 and 8b of the California publication reporting the above experiments, together with the accompanying discussions.

Other applications.—It is desirable briefly to view the advantages, in a study of the moisture conditions above a high water table, which result from capillary potential measurements. Referring to figure 15, measurements of the capillary potential at the points $P_1, P_2, \cdots P_n$ at elevations $h_1, h_2 \cdots h_n$ above the water table are represented by $\Psi_1, \Psi_2 \cdots \Psi_n$. If $h_1, h_2 \cdots h_n$ are measured in centimeters, they are numerically equal to the gravitational potentials $\varphi_1, \varphi_2 \cdots \varphi_n$ at the respective points. Suppose that $\Psi_1+h_1=\Psi_2+h_2= \cdots \Psi_n+h_n=$ constant. It would follow that the moisture distribution was in static equilibrium as represented in curve (a) of figure 15. Further, suppose that $\Psi_1+h_1>\Psi_2+h_2> \cdots \Psi_n+h_n \cdots$;* it then follows that the moisture distribution would be illustrated by curve (c) of figure 15 and that the moisture would move upward until an equilibrium of moisture distribution was reached. On the other hand, if $\Psi_1+h_1<\Psi_2+h_2< \cdots \Psi_n+h_n$, the moisture distribution would be represented by curve (b) of figure 15 and the moisture would move into the water table until equilibrium was established. Since there is no positive hydrostatic pressure above a water table it is evident that for vertical flow $\nabla\Phi$ of equation (40) would equal $\dfrac{(\Psi_1+h_1)-(\Psi_2+h_2)}{h_2-h_1}$ or $\dfrac{(\Psi_2+h_2)-(\Psi_3+h_3)}{h_3-h_2}$, etc. Hence, for any soil, the

* It is important here to remember that Ψ is negative and hence that a high numerical value results in a low absolute value.

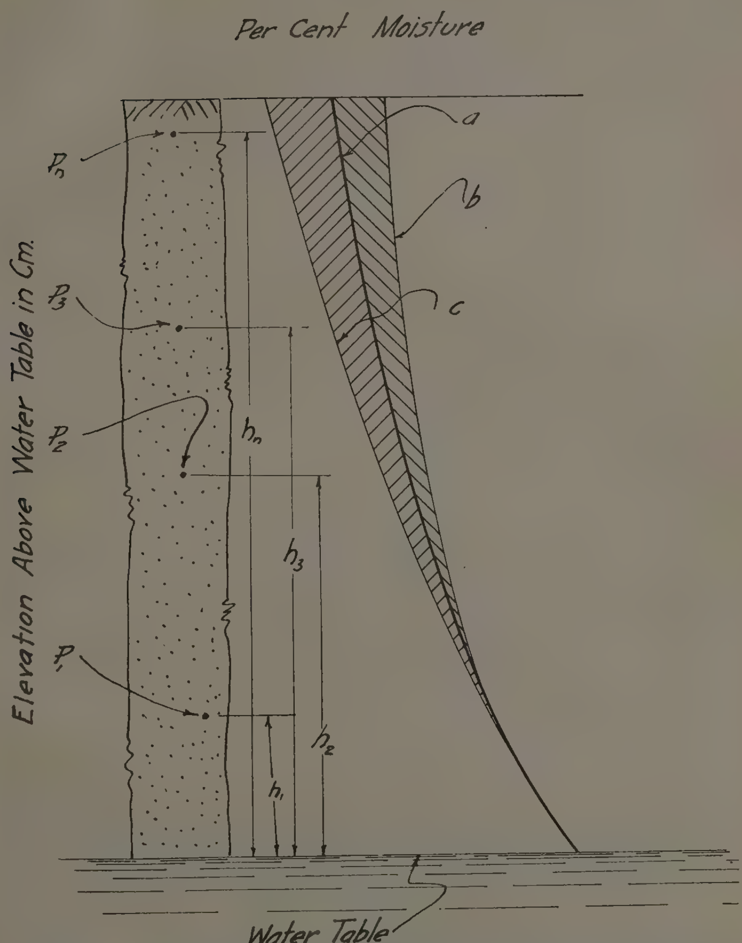

Fig. 15. The probable moisture distribution above a water table for three conditions, namely: *a*, equilibrium, in which the sum of potentials is constant, no moisture movement; *b*, Ψ is less than for equilibrium and moisture moves into water table; *c*, Ψ is greater than for equilibrium and moisture moves out of water table.

average capillary conductivity of which is known, a few measurements of capillary potential and of the average moisture content would make it possible quantitatively to determine, by use of equation (40), after a steady flow had been established, the loss from, or the contribution

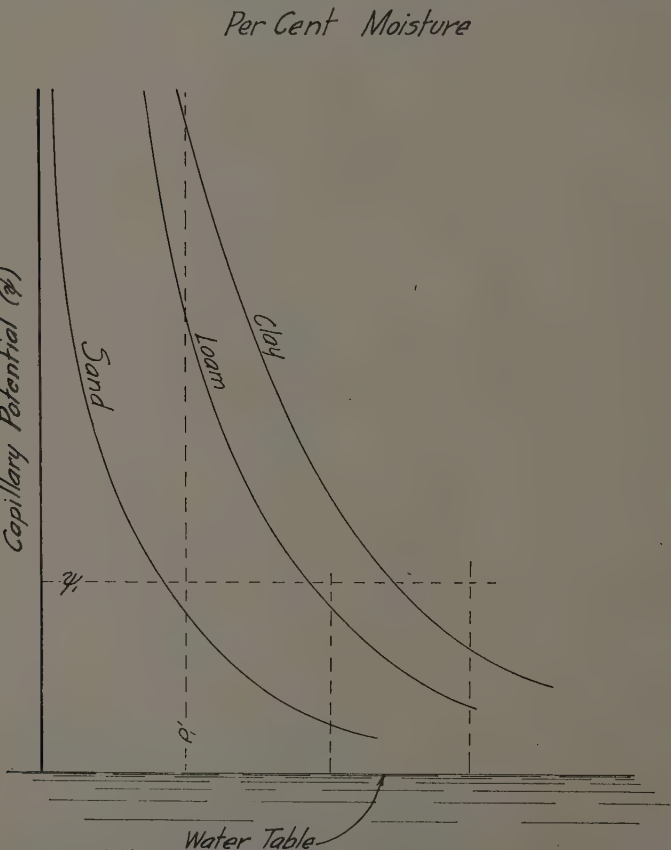

Fig. 16. The influence of soil texture on the probable relation of the capillary potential (Ψ) to the moisture content (ρ').

to, a water table resulting from the capillary stream, in a given period of time.

It appears that the quantitative relation of Ψ to ρ' is different for different types of soil. Work thus far at the Utah Agricultural Experiment Station indicates that for a given capillary potential Ψ the moisture

content is highest in a clay, medium in a loam, and lowest in a sand and, conversely, for a given ρ' the capillary potential Ψ is highest in a clay, medium in a loam, and lowest in a sand. These relations are illustrated in figure 16.

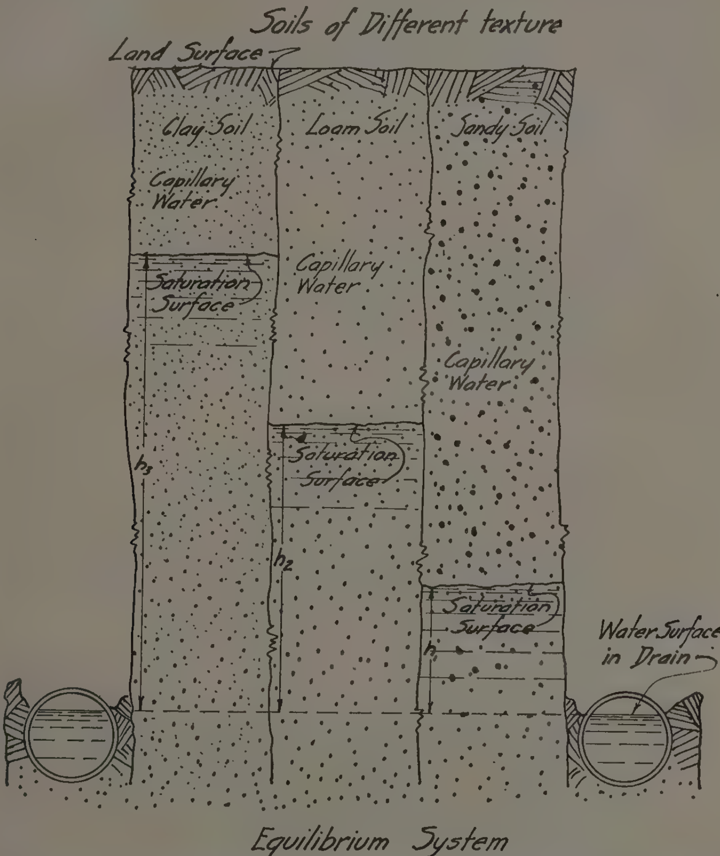

Fig. 17. The influence of soil texture on the probable differences in elevations of the water table and the surfaces of water saturation in the soil under equilibrium conditions.

The above facts have an important bearing on drainage. In the finer textured soils it is necessary to place drains comparatively deep in order to be sure that, in the upper few feet of soil, the capillary potential will be high and thus bring about a low moisture content. Moreover, both observations and experiment tend to show that very fine-textured soils

have such a high capillary potential with comparatively large percentages of capillary water that the soils actually remain saturated considerable distances above the water table as illustrated in figure 17. A very fine-textured soil thus holds the gravitational water in equilibrium above the water table a distance h_3; a medium-textured one a distance h_2; and a coarse-textured one a very small distance h_1, in which $h_3 > h_2 > h_1$.

The above examples suggest other possible applications of hydromechanics to irrigation and drainage problems. Recent applications by Linford[14] in a study of the relation of light to soil moisture phenomena seem to be very promising. The application of analytical methods to problems in irrigation and drainage, which are admittedly complex, is undoubtedly a more rational procedure than the pursuing of empirical methods without the guidance of fundamental principles. The heterogeneity of soils in reality increases rather than decreases the need for applying the laws of motion to these problems, despite the conditions of extreme variability sometimes encountered in soils in which measurements of soil moisture content or flow by any method are impracticable.

SUMMARY AND CONCLUSIONS

1. Knowledge of the laws which govern the movement of ground water and soil moisture is essential to its effective control.

2. The fundamental hydrodynamical equation of motion and the equation of continuity may be applied to irrigation and drainage problems.

3. For irrotational motion the velocity may be derived from a potential.

4. The driving forces in the formula for flow of water in open channels and in pipes are derivable from a potential.

5. Applications of hydrodynamics to ground-water and soil-moisture movement have been made by only a few investigators.

6. The capillary potential can be measured in the laboratory with the aid of porous cups by methods herein described.

7. The capillary potential Ψ is a function of the moisture content ρ'. Analysis of 84 determinations of the relation of Ψ to ρ' suggest that it may be represented by an equilateral hyperbola of the form $(\rho' - a)(\Psi + b) = c$.

8. The moisture distribution at equilibrium in a vertical soil column is not uniform but decreases with height above the water table. Such distribution qualitatively confirms the requirements for an equipotential moisture region.

ACKNOWLEDGMENTS

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LITERATURE CITED

[1] Briggs, Lyman J.
1899. The movement and retention of water in soils. U. S. Dept. Agr. Yearbook 1898: 399–404.

[2] Briggs, Lyman J.
1898. The mechanics of soil moisture. U. S. Dept. Agr. Bur. Soils Bul. **10**:1–24. figs. 1–7.

[3] Buckingham, Edgar
1907. Studies on the movement of soil moisture. U. S. Dept. Agr. Bur. Soils Bul. **38**:1–61. figs. 1–23.

[4] Coffin, Joseph George
1911. Vector analysis. 1–253. figs. 1–69. John Wiley & Sons, N. Y.

[5] Etcheverry, B. A.
1915. Irrigation practice and engineering—Conveyance of water. **2**:1–350. figs. 1–81. McGraw-Hill Book Co., N. Y.

[6] Gardner, Willard
1919. The movement of moisture in soil by capillarity. Soil Sci. **7**:314–315. figs. 1–3.

[7] Gardner, Willard
1920. The capillary potential and its relation to soil-moisture constants. Soil Sci. **10**:357–359. fig. 1.

[8] Gardner, Willard and John A. Widtsoe.
1921. The movement of soil moisture. Soil Sci. **11**:215–232. figs. 1–11.

[9] Houstoun, R. A.
1920. An introduction to mathematical physics. 1–197. figs. 1–75. Longmans, Green & Co., N. Y.

[10] Israelsen, O. W.
1918. Studies on capacities of soils for irrigation water. Jour. Agr. Res. **13**:1–36. figs. 1–14. pl. 1.

[11] Israelsen, O. W. and Frank L. West.
1922. Water-holding capacity of irrigated soils. Utah Agr. Exp. Sta. Bul. **183**:1–24. figs. 1–7.

[12] KEEN, B. A.
1924. On the mutual relationships in ideal soil. Jour. Agr. Sci. **14**:170–177. figs. 1–4.

[13] LAMB, HORACE
1916. Hydrodynamics. Fourth edition. 1–701. Cambridge University Press, Cambridge, England.

[14] LINFORD, LEON B.
1926. The relation of light to soil moisture phenomena. Soil Sci. **10**:233–251. figs. 1–6.

[15] MCLAUGHLIN, W. W.
1920. Capillary movement of soil moisture. U. S. Dept. Agr. Bul. **835**:1–69. figs. 1–10.

[16] MCLAUGHLIN, W. W.
1924. The capillary distribution of moisture in soil columns of small cross section. U. S. Dept. Agr. Bul. **1221**:1–122. figs. 1–7.

[17] SLICHTER, CHARLES S.
1898. Theoretical investigations of the motion of ground water. U. S. Geol. Survey 19th Ann. Rept. **2**:301–380.

[18] VEIHMEYER, F. J., O. W. ISRAELSEN, and J. P. CONRAD.
1924. The moisture equivalent as influenced by the amount of soil used in its determination. California Agr. Exp. Sta. Tech. Paper **16**:1–60. figs. 1–9. pl. 1.

[19] WIDTSOE, JOHN A. and W. W. MCLAUGHLIN.
1912. The movement of water in irrigated soils. Utah Agr. Exp. Sta. Bul. **115**:199–268. fig. 1. pls. 6.

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